HONORS DISCRETE MATHEMATICS - CS

Grade Level: 9

Credits: 2.5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 27, 2012

SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING
APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS
APPENDIX B: ASSESSMENT EVIDENCE
APPENDIX C: INTERDISCIPLINARY CONNECTIONS
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Introduction

Course Philosophy

This course is designed for mathematically talented students studying computer science. It encompasses logical reasoning, critical thinking, and problem solving. A variety of methods are incorporated to visually represent and organize data. The topics included are meant to build a foundation for future courses in discrete mathematics and computer science. Representing real-world scenarios with graphs and applying sound reasoning skills to solve problems are processes that are applicable to future academic studies and careers.

Course Description

Discrete mathematics for computer science students examines the concepts of number theory, set theory, and graph theory. Students will explore number systems other than our commonly used decimal system, and investigate the applications of such in the realm of computer science. Boolean algebra, sequences, recursion, iteration, and combinatorics will also be addressed. Students will be exposed to a wide variety of topics and guided through the process of learning to think abstractly. Representations such as tables, graphs, sets, mappings, and networks are employed as tools to organize information. The use of proper notation and vocabulary will be emphasized throughout the course.
### Course Map and Proficiencies/Pacing

#### Course Map

<table>
<thead>
<tr>
<th>Relevant Standards</th>
<th>Enduring Understandings</th>
<th>Essential Questions</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-Q2</td>
<td>In addition to the base 10 number system, there are various number systems present in everyday life.</td>
<td>What other number systems besides base 10 are useful in the study of computer science?</td>
<td>Diagnostic</td>
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<tr>
<td></td>
<td></td>
<td>What are some applications of the other number systems?</td>
<td>Formative</td>
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<td></td>
<td>What relationships exist among these different number systems?</td>
<td>Summative</td>
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<td></td>
<td></td>
<td>Give simple arithmetic problems in base 10. Students focus on place value, symbols used, and the process of adding or subtracting.</td>
<td>Practice problems in class, for homework and on quizzes that require arithmetic in binary, octal, and hexadecimal number systems</td>
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<tr>
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<td></td>
<td>Unit test including questions that require converting numbers from one base to another</td>
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<tr>
<td>S-CP1</td>
<td>Sets and their properties are useful in organizing and comparing both simple and complex elements.</td>
<td>How are sets useful in organizing data?</td>
<td>Homework, class work, and quiz questions which utilize various properties of sets in general, including proper notation and vocabulary</td>
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<tr>
<td></td>
<td></td>
<td>In what ways can sets be represented visually?</td>
<td>Unit test including disproofs and proofs of properties</td>
</tr>
<tr>
<td>N-Q2</td>
<td>Logical symbols and reasoning can be used to represent statements and arguments in an efficient manner.</td>
<td>What are the symbols commonly used in logic?</td>
<td>What are some methods of organizing information and statements in the realm of logic?</td>
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<td>---------------------------------------------------------------------------------</td>
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<tr>
<td>F-BF2</td>
<td>Sequences can be used to represent patterns that exist in mathematics and the world around us.</td>
<td>How are explicit formulas used to represent sequences?</td>
<td>What are some applications of sequences?</td>
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<tr>
<td>F-IF3</td>
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<tr>
<td>F-LE2</td>
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<tr>
<td>F-IF1</td>
<td>Relations and functions are defined on sets of numbers and have extensive mathematical applications.</td>
<td>What are some properties of relations?</td>
<td>What are some properties of functions?</td>
</tr>
<tr>
<td>F-IF2</td>
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<td>F-IF3</td>
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<td>F-IF5</td>
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<td>F-BF1a</td>
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<td>F-BF1c</td>
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<td>F-BF4</td>
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<tr>
<td>Standard</td>
<td>Description</td>
<td>Questions</td>
<td>Activities</td>
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<td>-----------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
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</tbody>
</table>
| S-CP9    | Algorithms can effectively and efficiently be used to quantify and interpret discrete information. | What algorithms are used in conjunction with counting principles?  
How do you determine which counting principle is used to solve a problem?  
How are combinations related to subsets of a set? | Students will determine the number of outcomes of a situation using a tree diagram.  
Practice in class, at home, and quiz questions involving counting principle, permutations, combinations, and Pascal's Triangle | Unit test with some probability applications                                                   |
| N-Q2     | Optimization is the process of finding the best solution within given constraints. | How can visual tools such as trees, networks, and circuits be used to provide information?  
What is the significance of a connected graph?  
How can vertex-edge graphs be used to model relationships and solve a variety of problems? | Given a communication matrix, draw the corresponding network.  
Questions in which information is extracted from a network or circuit, as well as questions that require students to draw trees, paths, or circuits to represent given situations | Unit test including applications of graph theory                                               |
| N-VM6    |                                                                 |                                                                                                                             |                                                                                                |                                                                                                |
## Proficiencies and Pacing

<table>
<thead>
<tr>
<th>Unit Title</th>
<th>Unit Understanding(s) and Goal(s)</th>
<th>Recommended Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit 1: Number Systems</strong></td>
<td>In addition to the base 10 number system, there are various number systems present in everyday life.</td>
<td>4 weeks</td>
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<tr>
<td></td>
<td>At the conclusion of this unit, students will be able to:</td>
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<tr>
<td></td>
<td>1. compute values in and convert between various number systems.</td>
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<td></td>
<td>2. perform operations and simplify expressions using Boolean algebra.</td>
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<td></td>
<td>3. interpret diagrams involving digital electronics.</td>
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<td></td>
<td>4. evaluate operators with bit strings.</td>
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<tr>
<td><strong>Unit 2: Logic and Reasoning</strong></td>
<td>Logical symbols and reasoning can be used to represent statements and arguments in an efficient manner.</td>
<td>2 weeks</td>
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<tr>
<td></td>
<td>At the conclusion of this unit, students will be able to:</td>
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<tr>
<td></td>
<td>1. use truth tables to determine truth values of compound statements.</td>
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<td></td>
<td>2. determine whether two expressions are logically equivalent.</td>
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<tr>
<td></td>
<td>3. construct logical arguments using laws of detachment, syllogism, tautology, and contradiction.</td>
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<td></td>
<td>4. determine the validity of an argument.</td>
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<td></td>
<td>5. apply universal and existential quantifiers.</td>
<td></td>
</tr>
<tr>
<td><strong>Unit 3: Number Theory</strong></td>
<td>In addition to the base 10 number system, there are various number systems present in everyday life.</td>
<td>1 week</td>
</tr>
<tr>
<td></td>
<td>At the conclusion of this unit, students will be able to:</td>
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<tr>
<td></td>
<td>1. use the binary operations of div and mod to express a quotient and a remainder.</td>
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<tr>
<td></td>
<td>2. determine the greatest common divisor of two integers using Euclid's Algorithm.</td>
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</tr>
<tr>
<td></td>
<td>3. perform modular arithmetic operations and use them to solve equations.</td>
<td></td>
</tr>
</tbody>
</table>
| Unit 4: Sequences and Recursion | Sequences can be used to represent patterns that exist in mathematics and the world around us.  
At the conclusion of this unit, students will be able to:  
1. find explicit and recursive formulas for arithmetic, geometric, and other sequences and use these formulas to determine a specific term or term number.  
2. convert between a series and its sigma notation or between a product and its pi notation.  
3. find partial sums of arithmetic and geometric series and find sums of convergent infinite series.  
4. generate other recursive sequences such as factorials and the Fibonacci sequence.  
5. apply recurrence, recursive, and iterative thinking to solve problems. | 1.5 weeks |
| --- | --- | --- |
| Unit 5: Set Theory | Sets and their properties are useful in organizing and comparing both simple and complex elements.  
At the conclusion of this unit, students will be able to:  
1. recognize and apply the concepts of set equality, subset, and null set.  
2. perform set operations such as union, intersection, difference, complement, and cross-product.  
3. use Venn diagrams to depict and explore relationships among sets. | 1.5 weeks |
| Unit 6: Functions and Relations | Relations and functions are defined on sets of numbers and have extensive mathematical applications.  
At the conclusion of this unit, students will be able to:  
1. recognize and evaluate functions and their properties, such as one-to-one and onto functions.  
2. write the inverse of a function and perform the composition of functions.  
3. express a relation and its inverse.  
4. use concepts of reflexivity, symmetry, and transitivity to establish that a relation is an equivalence. | 2.5 weeks |
| Unit 7: Counting and Probability | Algorithms can effectively and efficiently be used to quantify and interpret discrete information.  
At the conclusion of this unit, students will be able to:  
1. apply fundamental counting principles to solve combinatorial and probability problems.  
2. compute permutations of set with non-repeated and with repeated elements.  
3. compute combinations and relate them to Pascal's Triangle and the Binomial Theorem. | 2 weeks |
| Unit 8: Graph Theory | Optimization is the process of finding the best solution within given constraints.  
At the conclusion of this unit, students will be able to:  
1. use graphs to model and solve problems such as shortest paths, circuits, vertex coloring, critical paths, routing, and scheduling problems.  
2. convert from a graph to a matrix and vice versa.  
3. use directed graphs, spanning trees, rooted trees, binary trees, or decision trees to solve problems.  
4. use bin-packing techniques to solve optimization problems. | 2.5 weeks |
# Honors Discrete Math - CS Unit 01

## Unit Plan

### Enduring Understandings:
In addition to the base 10 number system, there are various number systems present in everyday life.

### Essential Questions:
- What other number systems besides base 10 are useful in the study of computer science?
- What are some applications of the other number systems?
- What relationships exist among these different number systems?

### Unit Goals:
At the conclusion of this unit, students will be able to:
1. compute values in and convert between various number systems.
2. perform operations and simplify expressions using Boolean algebra.
3. interpret diagrams involving digital electronics.
4. evaluate operators with bit strings.

### Recommended Duration: 4 weeks

<table>
<thead>
<tr>
<th>Guiding/Topical Questions</th>
<th>Content/Themes/Skills</th>
<th>Resources and Materials</th>
<th>Suggested Strategies</th>
<th>Suggested Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the basic steps involved in performing addition and subtraction of two digit numbers in the base ten number system?</td>
<td>Place value</td>
<td>Simple arithmetic problems in base 10</td>
<td>Guide students to understand the process behind the arithmetic, emphasizing place value and powers of 10, rather than the rote operations.</td>
<td>Perform arithmetic in the decimal number system and explain the process behind each step.</td>
</tr>
<tr>
<td></td>
<td>Carrying or borrowing a group of ten</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How can these processes be transferred to numbers with bases other than ten?</td>
<td>Place value using a base other than ten</td>
<td>Worksheets of practice problems</td>
<td>Perform arithmetic such as addition and subtraction in the binary, octal, and hexadecimal number systems.</td>
<td>Perform arithmetic with numbers given in several different bases in the same problem.</td>
</tr>
<tr>
<td></td>
<td>Carrying or borrowing a group of 2, 8, or 16</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>How are truth tables used to organize information in Boolean algebra?</td>
<td>Truth table</td>
<td>Textbook</td>
<td>Create truth tables to explore the properties of Boolean algebra.</td>
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</tr>
<tr>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
<td>Given a diagram of digital electronics, translate it into Boolean algebra and determine its truth value.</td>
<td></td>
</tr>
<tr>
<td>Logically equivalent expressions</td>
<td>Logically equivalent expressions</td>
<td>Logically equivalent expressions</td>
<td>Logically equivalent expressions</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How are expressions simplified in Boolean algebra?</th>
<th>Properties and identities in Boolean algebra</th>
<th>Reference sheet which summarizes properties and identities of Boolean algebra</th>
<th>Engage in guided practice simplifying expressions, followed by partner activity: create an expression in Boolean algebra; give it to your partner to simplify; switch back to correct.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Properties and identities in Boolean algebra</td>
<td>Properties and identities in Boolean algebra</td>
<td>Simplify an expression in Boolean algebra and justify each step with a property or identity.</td>
</tr>
<tr>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
</tr>
<tr>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What operations can be performed using bit string analysis?</th>
<th>Logical operators NOT, AND, OR</th>
<th>Logical operators NOT, AND, OR</th>
<th>Simplify expressions involving bit strings of varying lengths by evaluating operators, SHIFTs, and CIRCs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
<td>Logical operators NOT, AND, OR</td>
</tr>
<tr>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
<td>LSHIFT, RSHIFT, LCIRC, RCIRC</td>
</tr>
</tbody>
</table>

MA.9-12.HSN-Q.2 Define appropriate quantities for the purpose of descriptive modeling.

**Differentiation**

Extensive review of arithmetic processes in the decimal number system will aid in the transfer of these processes to systems with bases other than 10. Colored pencils can be used to mark the different place values in the number system problems or to mark the diagrams of digital electronics. Both the computer number systems and the Boolean algebra problems lend themselves to varying degrees of difficulty, so that students may be given problems of different complexity throughout the classroom.
Technology

Students can write a computer program that receives the truth value of two variables as input. The output of the program will be the truth value of a Boolean algebra expression.

College and Workplace Readiness

Differently based number systems are used in many global occupations. Scientists utilize base 10 systems in calculations worldwide. Electricians also utilize another base number system for circuits. Binary systems are still used in computer science. Students can research various based number systems and how they are used in these careers.
Honors Discrete Math - CS Unit 02

Unit Plan

Enduring Understandings:
Logical symbols and reasoning can be used to represent statements and arguments in an efficient manner.

Essential Questions:
What are the symbols commonly used in logic?
What are some methods of organizing information and statements in the realm of logic?
How are arguments proven or disproven?

Unit Goals:
At the conclusion of this unit, students will be able to:
1. use truth tables to determine truth values of compound statements.
2. determine whether two expressions are logically equivalent.
3. construct logical arguments using laws of detachment, syllogism, tautology, and contradiction.
4. determine the validity of an argument.
5. apply universal and existential quantifiers.

Recommended Duration: 2 weeks

<table>
<thead>
<tr>
<th>Guiding/Topical Questions</th>
<th>Content/Themes/Skills</th>
<th>Resources and Materials</th>
<th>Suggested Strategies</th>
<th>Suggested Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>How can truth tables aid in determining the value of a compound expression?</td>
<td>Truth table</td>
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<td></td>
<td>Compound statements involving not, and, or, exclusive or, but, neither-nor</td>
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<td></td>
<td>Tautology</td>
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<td>Contradiction</td>
<td></td>
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<td></td>
<td>conditional statement, converse, inverse, and contrapositive</td>
<td>Worksheets of expressions for which to create truth tables</td>
<td>Given compound statements, construct truth tables for each.</td>
<td>Given statements, translate them into compound statements of logic, construct truth tables, and determine which values make the compound statement true (or false).</td>
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</tbody>
</table>

Given compound statements, determine which one(s) are tautologies, which one(s) are contradictions. Explain why.
<table>
<thead>
<tr>
<th>What does it mean for statements to be logically equivalent?</th>
<th>Properties and identities of Boolean algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>What makes an argument valid?</td>
<td>Argument</td>
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<td>Argument form</td>
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<td>Syllogism</td>
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<td>Modus ponens</td>
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<td>Modus tollens</td>
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<td>Contradiction rule</td>
</tr>
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<td>What is the difference between the phrases &quot;for every x&quot; and &quot;there is an x such that&quot;?</td>
<td>Statement</td>
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<td>Predicate</td>
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<td>Quantifiers</td>
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<td>Universal quantifier</td>
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<td>Method of exhaustion</td>
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<td>Existential quantifier</td>
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<td>Necessary</td>
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<td></td>
<td>Sufficient</td>
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</tbody>
</table>

- **Text**: Review sheet of Boolean algebra properties and identities
- **Worksheet of compound statements**
- **Construct truth tables for compound statements and compare the results. Analyze which statements produce the same truth values.**
- **Determine whether two statements are logically equivalent; justify your answer.**
- **Given a compound statement, create another statement that is logically equivalent to it.**

- **Text**: Multimedia presentation of different argument forms
- **Worksheets of arguments**
- **Given several arguments in similar form, look for a pattern and generalize the form using variables.**
- **Given an argument form in symbolic form, write a truth table, label the premises and conclusion, and determine whether the argument form is valid.**
- **Given an argument in word form, rewrite it in logical form using symbols. Determine whether the argument is valid or invalid and state the rule of inference or the error that is made.**

- **Text**: Worksheet of statements to translate into logical form
- **Index cards with statements out of order; rearrange them and draw a valid conclusion.**
- **Given a quantified statement, find a counterexample to show it is false.**
- **Translate word sentences into quantified statements.**
- **From a set of statements, identify which ones are equivalent.**
- **Given an argument in word form, translate it into logical form using quantifiers. Determine whether it is valid or invalid.**
- **Use diagrams to test the validity of an argument.**

**MA.9-12.HSN-Q.2** Define appropriate quantities for the purpose of descriptive modeling.
Differentiation

The abstract nature of this unit can be supplemented with tangible objects such as index cards with statements on them. These can be used for matching equivalent statements or arranging the statements of an argument in a proper sequence. Colored pencils or highlighters may aid in marking significant vocabulary that determines which quantifier should be used when translating a statement.

Technology

Students will compare the vocabulary and symbolic notation used in this unit with the current language being used in their programming course. Students will demonstrate proficiency on the relationship between the structure of a logical argument and the structure of a computer program.

College and Workplace Readiness

Students successful in this unit will be better prepared for occupations that require communication skills such as the construction of strong, clear and persuasive arguments. Opportunities that exist include attorneys, or any other professional presenters. Students can complete a persuasive argument essay to demonstrate these skills.
Honors Discrete Math - CS Unit 03

Unit Plan

Enduring Understandings:
In addition to the base 10 number system, there are various number systems present in everyday life.

Essential Questions:
What other number systems besides base 10 are useful in the study of computer science?
What are some applications of the other number systems?

Unit Goals:
At the conclusion of this unit, students will be able to:
1. use the binary operations of div and mod to express a quotient and a remainder.
2. determine the greatest common divisor of two integers using Euclid's Algorithm.
3. perform modular arithmetic operations and use them to solve equations.

Recommended Duration: 1 week

<table>
<thead>
<tr>
<th>Guiding/Topical Questions</th>
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<th>Suggested Strategies</th>
<th>Suggested Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the difference between a quotient and a remainder in a division problem?</td>
<td>Quotient, Remainder, Div and mod operators</td>
<td>Worksheets of problems in arithmetic and modular form</td>
<td>Evaluate several arithmetic expressions, identifying the quotient and remainder, and use them to discover a pattern that can be translated into notation using the binary operators of div and mod.</td>
<td>Investigate how at least two different programming languages use the mod and div (or int) operations. Describe similarities and differences to the operators presented in this class.</td>
</tr>
<tr>
<td>Why is it useful to calculate the greatest common divisor of a pair of integers?</td>
<td>Greatest common divisor</td>
<td>Text</td>
<td>Begin with arithmetic problems to determine the greatest common divisor by listing all divisors of a pair of integers. Explore Euclid's algorithm as presented in two different manners, one using quotient-remainder theorem and one using mod notation. Draw parallels between the two presentations, showing how they mean the same thing.</td>
<td>Write a computer program that calculates the greatest common divisor of two integers using Euclid's algorithm.</td>
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</tr>
<tr>
<td>How can modular arithmetic be used to reduce computations with large numbers to computations involving smaller ones?</td>
<td>Modular arithmetic</td>
<td>Text</td>
<td>Evaluate expressions involving modular arithmetic with a variety of bases. Use modular arithmetic to rewrite the equations in a system to involve smaller coefficients.</td>
<td>Solve practical problems using modular arithmetic such as calculating greatest common factor or creating error detection codes (as in a universal product code). Find everyday occurrences of modular arithmetic (e.g. clock mod 12).</td>
</tr>
</tbody>
</table>

**MA.9-12.HSN-Q.2** Define appropriate quantities for the purpose of descriptive modeling.
**Differentiation**

Partner work may increase student understanding of div, mod, and modular arithmetic. Each student can list divisors of an integer and then compare to see which ones they have in common. This can lead to the discussion of finding the greatest common divisor and the need for a shorter method to find one.

For modular arithmetic, a jigsaw approach may be taken in which each group of students learns a specific operation in modular arithmetic. New groups are then formed so that each group has a specialist in each operation. Students teach each other the operation they learned in the first group.

**Technology**

Students will write a computer program using their current programming language to demonstrate modular arithmetic, such as the Chinese Remainder Theorem, or Euclid's algorithm.

**College and Workplace Readiness**

Computer programmers, electrical engineers, and cryptographers utilize number theory. Applications include creating computer passwords and electronic commerce. Students can write a program to simplify imaginary numbers raised to a power.
### Honors Discrete Math - CS Unit 04

#### Unit Plan

**Enduring Understandings:**
Sequences can be used to represent patterns that exist in mathematics and the world around us.

**Essential Questions:**
How are explicit formulas used to represent sequences?
What are some applications of sequences?
What are some instances of recursive functions in computer science?

**Unit Goals:**
At the conclusion of this unit, students will be able to:
1. find explicit and recursive formulas for arithmetic, geometric, and other sequences and use these formulas to determine a specific term or term number.
2. convert between a series and its sigma notation or between a product and its pi notation.
3. find partial sums of arithmetic and geometric series and find sums of convergent infinite series.
4. generate other recursive sequences such as factorials and the Fibonacci sequence.
5. apply recurrence, recursive, and iterative thinking to solve problems.

**Recommended Duration:** 1.5 weeks

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<tbody>
<tr>
<td>How can a pattern of a sequence be determined so that the sequence can be continued?</td>
<td>Arithmetic sequence</td>
<td>Worksheets of sequences, identify the pattern and write the next two terms</td>
<td>List a sequence of numbers that represents the number of ancestors there are going back 1, 2, 3, 4, and 5 generations. Write a rule to find the number of ancestors for k generations.</td>
<td>Given two different explicit formulas, write the first four terms of each and compare the results. Will the formulas produce exactly the same sequence? Write a sequence that is neither arithmetic nor geometric. Then provide an explicit formula for the sequence.</td>
</tr>
<tr>
<td></td>
<td>Geometric sequence</td>
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<td>Factorials</td>
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<td></td>
<td>Fibonacci Sequence</td>
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</tbody>
</table>

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**Essential Questions:**
How are explicit formulas used to represent sequences?
What are some applications of sequences?
What are some instances of recursive functions in computer science?
What shorthand can be used to abbreviate a series or a product of a sequence?

- Explicit formula
- Sigma notation for summation
- Pi notation for product

Handouts for practice with sigma notation and pi notation

Engage in guided practice with sigma notation, converting from a series to sigma and vice versa. Expand into representing a product of a sequence with pi notation.

Calculate summations and products from sigma and pi notation.

Given a series in expanded form, write two possible versions of sigma notation (or pi notation if it is a product) to represent it.

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How can recursively defined functions be useful?

- Recursive formula for a sequence
- Using the solutions to smaller scale problems to derive a solution to a larger problem

Tower of Hanoi wooden puzzle

Worksheet of computation and application problems

Use the Tower of Hanoi to demonstrate recursive thinking and mathematical induction, both of which use solutions to simpler problems to obtain a solution to a larger problem.

Solve problems involving a recursive formula for compound interest and the Fibonacci sequence.

How can iteration aid in deriving an explicit formula from a recursive one?

- Recursive versus explicit iteration process

Worksheet of formulas to convert from recursive to explicit rules

Engage in guided and independent practice with converting recursive formulas to explicit ones.

Solve application problems by first writing a recurrence relation and then using iteration to find an explicit formula.

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**Differentiation**

Students can use a wooden Tower of Hanoi puzzle to experience the steps necessary to move the disks according to the rules of the game. This hands-on experience can bring to life the concept of solving smaller problems (start with 1 disk, then 2, then 3, etc.) to develop a strategy for solving a larger problem. This concept can be repeated with a coin problem in which the penny is replaced with a 3 cent piece. Students can use chips or blocks to represent a 3 cent piece and a 5 cent piece and see what values of change are and are not possible to make. Using the manipulatives will make it easier to solve several simpler problems before attempting a large one. In this way the concepts of pattern, induction, and recursion are made more tangible.

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MA.9-12.HSF-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

MA.9-12.HSF-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

MA.9-12.HSF-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Technology

Students can utilize their graphing calculators in sequential mode to graph sequences and series. They can also utilize the sequence and sum functions to evaluate series. They can also explore both recursive and explicit rules on the graphing calculator.

College and Workplace Readiness

Students may complete a close reading activity analyzing the use of sequences and series to determine retirement savings based on recurring payments once they have started their careers. Students may also explore the use of recursive functions in other contexts. Physicists, for example, utilize a recursive program to calculate motion.
Honors Discrete Math - CS Unit 05

Unit Plan

Enduring Understandings:
Sets and their properties are useful in organizing and comparing both simple and complex elements.

Essential Questions:
How are sets useful in organizing data?
In what ways can sets be represented visually?
What are some commonly used sets of numbers?

Unit Goals:
At the conclusion of this unit, students will be able to:
1. recognize and apply the concepts of set equality, subset, and null set.
2. perform set operations such as union, intersection, difference, complement, and cross-product.
3. use Venn diagrams to depict and explore relationships among sets.

Recommended Duration: 1.5 weeks

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<tr>
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<tbody>
<tr>
<td>What sets of numbers are commonly used in problem solving?</td>
<td>Real number system</td>
<td>Handout with equations to solve and columns in which to solve them over different sets of numbers</td>
<td>Draw a Venn diagram of the set of real numbers and the subsets. Practice solving equations over different sets of numbers.</td>
<td>Solve equations over the natural, whole, integer and real number sets.</td>
</tr>
</tbody>
</table>
| How can sets of data be compared to one another? | Set-builder notation  
Subset, proper subset  
Venn diagram  
Empty set  
Disjoint sets | Worksheet of sets of data represented in set-builder notation as well as in diagrams | Given sets of data, identify any relationships among the sets (common elements, size of set, etc). | Given several sets of data, determine which ones are subsets, proper subsets, and disjoint sets. |
|---|---|---|---|---|
| What operations can be performed on a set? | Union  
Intersection  
Difference (relative complement)  
Complement  
Cartesian product (cross-product)  
Set Identities and Properties | Worksheet of Venn diagrams and sets | Given a Venn diagram, shade the region that represents given operations, such as A U B.  
Use the Set Identities to simplify expressions. | Given two sets, find their union, intersection, difference, and Cartesian product. |

**Differentiation**

Sets can be represented with three-dimensional objects, as elements listed on index cards, or as abstract items in a list. The use of color (red and blue blocks, colored index cards, colored pencils or markers) can aid in comparing sets. Making the objects tangible can make it easier to bridge into representing the sets with proper notation and operations.
Technology
Students can create a multimedia presentation that involves Venn diagrams to represent the relationship between sets of real-life data.

College and Workplace Readiness
Students will prepare for careers that require data-driven decision-making and methods of organizing data, such as educators and business administrators. Students can gather data, organize it, and analyze it to make a decision.
Honors Discrete Math - CS Unit 06

Unit Plan

Enduring Understandings:
Relations and functions are defined on sets of numbers and have extensive mathematical applications.

Essential Questions:
What are some properties of relations?
What are some properties of functions?
What is the significance of an equivalence relation?

Unit Goals:
At the conclusion of this unit, students will be able to:
1. recognize and evaluate functions and their properties, such as one-to-one and onto functions.
2. write the inverse of a function and perform the composition of functions.
3. express a relation and its inverse.
4. use concepts of reflexivity, symmetry, and transitivity to establish that a relation is an equivalence.

Recommended Duration: 2.5 weeks

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<tbody>
<tr>
<td>What are some examples of functions?</td>
<td>Domain</td>
<td>Worksheet of relations in various forms; determine if each is a function</td>
<td>Given relations in various forms (such as a mapping, list, table, formula, or set notation), determine whether or not each relation is a function.</td>
<td>Given a sequence, write a function for it.</td>
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<td>Given problems in set notation, evaluate each function.</td>
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<td>One-to-one</td>
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<td>Write three examples of functions in the world around us. Describe what makes it a function and then express it as a formula, mapping, or in set notation.</td>
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<td>Onto</td>
<td>Textbook</td>
<td>Investigate mod and div as functions.</td>
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<tr>
<td>How can the inverse of a function be useful?</td>
<td>Inverse function</td>
<td>Handout with functions represented in numerous ways</td>
<td>Given a function, determine its inverse. Then determine whether the inverse is a function. Given two equations, use composition of functions to show that they are inverse functions of each other. Briefly examine exponential functions and logarithmic functions as one-to-one functions and inverse functions.</td>
<td>Determine whether a function is one-to-one and onto, then write the inverse of the function. Describe an everyday function and its inverse.</td>
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<tr>
<td>What makes a relation an equivalence relation?</td>
<td>Relation notation xRy</td>
<td>Examples of relations to be tested for equivalence</td>
<td>Given a relation, determine whether it is reflexive, symmetric, and transitive. Provide a counterexample as appropriate.</td>
<td>Given a relation, determine whether the following is true or false. If R is reflexive, then the inverse of R is reflexive. Repeat for symmetric and transitive.</td>
</tr>
</tbody>
</table>

MA.9-12.HSF-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph off is the graph of the equation y = f(x).

MA.9-12.HSF-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.HSF-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

MA.9-12.HSF-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

MA.9-12.HSF-BF.1.a Determine an explicit expression, a recursive process, or steps for calculation from a context.

MA.9-12.HSF-BF.1.c Compose functions.

MA.9-12.HSF-BF.4 Find inverse functions.

**Differentiation**

Diagrams of function machines can be used to represent both simple and complex functions, such as mod, div, exponential and logarithmic functions. Functions can be expressed in set notation or in mapping form, to allow students to see different arrangements of the data. Drawing arrows from domain to range can aid in determining whether or not a relation is a function.
Technology

Students can utilize graphing calculators to demonstrate the relationship between the graph of a function and its inverse. Students can also create a multimedia presentation illustrating the concept of a function and how to derive its inverse. The presentation can also include the composition of functions to prove they are inverses of each other. The graphs can also be displayed to demonstrate the existing symmetry. A real-world example of a function can also be included in the multimedia presentation.

College and Workplace Readiness

Medical doctors must be excellent problem-solvers, and must utilize functions and the principles behind them. Teachers can create an activity with fictitious patient data and fictitious dosage information for a specific medication. Students must act as a physician and create patient prescriptions utilizing functions against specific variables relevant to the patient.
Honors Discrete Math - CS Unit 07

Unit Plan

Enduring Understandings:
Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
What algorithms are used in conjunction with counting principles?
How do you determine which counting principle is used to solve a problem?
How are combinations related to subsets of a set?

Unit Goals:
At the conclusion of this unit, students will be able to:
1. apply fundamental counting principles to solve combinatorial and probability problems.
2. compute permutations of set with non-repeated and with repeated elements.
3. compute combinations and relate them to Pascal's Triangle and the Binomial Theorem.

Recommended Duration: 2 weeks

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<tbody>
<tr>
<td>How many different PINs can be created by using four alphanumeric symbols, allowing repetition?</td>
<td>List method</td>
<td>Coins</td>
<td>List all outcomes or calculate the number of outcomes of problems involving drawing a card from a standard deck, tossing coins, or rolling dice.</td>
<td>Calculate the number of hexadecimal numbers that can be made that start with certain digits, end with certain digits, and are a given number of digits long. Compute the number of different PINs that are represented by one sequence of keys on a keypad, such as 2133.</td>
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<tr>
<td></td>
<td>Tree diagram</td>
<td>Dice</td>
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<td></td>
<td>Fundamental Counting Principle (The Multiplication Rule)</td>
<td>Cards</td>
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<td>Worksheets of practice problems</td>
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<tr>
<td>In how many ways can six people be seated around a circular table?</td>
<td>Permutations</td>
<td>Colored blocks</td>
<td>Use diagrams, blocks, coins, or other manipulatives to examine smaller problems in order to develop a pattern for solving larger problems.</td>
<td>Calculate the number of permutations of n objects in a row and in a circle.</td>
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<tr>
<td>How many subsets does a set with n elements have in all?</td>
<td>Subsets</td>
<td>Manipulatives such as blocks or coins</td>
<td>Partner activity: Given a set of 4 objects (such as four different coins), create all of the subsets possible and record the results in a table. Different pairs of students have different numbers of items in original sets. Compare the results as a class. Draw conclusions as to how many subsets there are in all for a set of n elements. Relate this Pascal's Triangle and to binomial expansion.</td>
<td>Find the number of diagonals in an n-gon. Write the first five terms of a binomial expansion to a power greater than ten.</td>
</tr>
</tbody>
</table>

MA.9-12.HSS-CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

**Differentiation**

Manipulatives such as cards, coins, dice and blocks can be used for students to physically create arrangements of items as well as subsets. Results from problems with small amounts of elements such as 2, 3, 4, or 5 items, can be recorded in a table and discussed as a class. Diagrams of arrangements in a row or in a circle can be pre-made for students to fill in with blocks or by coloring them in.
Technology

Students can utilize graphing calculators to compute the number of combinations and/or permutations of a set. Student can write a program using their current programming language to explore PIN numbers. Given four digits, students will determine the number of different PINs represented by those four digits on a keypad.

College and Workplace Readiness

Students can create a program to accept and store a list of daily exchange rates of various nations and extrapolate rate change trends. Market researchers benefit from a strong mathematical background. They often create surveys and interviews that provide information that is accurate and meaningful. Market researchers also need to have the skills to effectively analyze data. Much of the information is eventually presented in the form of charts and graphs. The data should reveal such things as market trends and customer preferences. Other related careers include sales manager, economist, and product manager.
Honors Discrete Math - CS Unit 08

Unit Plan

Enduring Understandings:
Optimization is the process of finding the best solution within given constraints.

Essential Questions:
How can visual tools such as trees, networks, and circuits be used to provide information?
What is the significance of a connected graph?
How can vertex-edge graphs be used to model relationships and solve a variety of problems?

Unit Goals:
At the conclusion of this unit, students will be able to:
1. use graphs to model and solve problems such as shortest paths, circuits, vertex coloring, critical paths, routing, and scheduling problems.
2. convert from a graph to a matrix and vice versa.
3. use directed graphs, spanning trees, rooted trees, binary trees, or decision trees to solve problems.
4. use bin-packing techniques to solve optimization problems.

Recommended Duration: 2.5 weeks
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<tbody>
<tr>
<td>Given a vertex-edge diagram, how can it be determined by inspection whether or not the figure can be traced without lifting the pencil and without duplicating a side?</td>
<td>Graph, Vertices, Edges, Valence of a vertex, Network, Connected graph, Circuit, Vertex coloring</td>
<td>Worksheet of diagrams (trees, networks, circuits) and problems to be translated into similar diagrams</td>
<td>Given several diagrams, determine which can be traced without lifting the pencil and without retracing any edges. Then determine which ones also begin and end at the same point. Identify the common characteristics in order to discover the pattern. Explore the Traveling Salesman Problem and other Hamiltonian circuits. Represent a scheduling problem (such as final exams) with a vertex-edge graph. Determine the minimum number of colors needed for vertex coloring.</td>
<td>Given a circuit, determine if it is an Euler circuit, a Hamiltonian circuit, or neither. Justify your answer. Draw a network that represents the streets of a town (snow removal, mail delivery, etc.). Eulerize the circuit.</td>
</tr>
<tr>
<td>How can a matrix be used to represent a communication digraph?</td>
<td>Directed graph (digraph), Matrix, Matrix multiplication</td>
<td>Handout with examples of communication matrices and/or networks, as well as tournament play</td>
<td>Given a communication network, write a matrix for direct communication and a matrix that represents one-relay communications. Given a matrix, draw its corresponding communication network.</td>
<td>Represent tournament play with a matrix and a network.</td>
</tr>
</tbody>
</table>
| How can trees aid in organizing and analyzing information? | Binary tree  
Rooted tree  
Spanning tree | Multimedia presentation of trees  
Practice problems with diagrams of different types of trees | Given a rooted tree, identify levels, height, parents, and children.  
Determine whether a tree is a binary tree, and if so, whether it is a full binary tree. Demonstrate examples and applications of each type of tree. |
| --- | --- | --- | --- |
| What are some scenarios that can be represented by bin-packing? | Weighted objects  
Bin-packing  
First-fit method  
Next-fit method  
Worst-fit method | Examples of scenarios that can be represented by weights and bins | Describe each of the methods for bin-packing and the advantages for each. Demonstrate examples of each, using the same data and comparing the outcomes. |
| | | | Determine which bin-packing arrangement is appropriate for a given problem. |

MA.9-12.HSN-Q.2 Define appropriate quantities for the purpose of descriptive modeling.
MA.9-12.HSN-VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

**Differentiation**

Drawings of trees, networks, circuits, and bins can be pre-made and provided to students. Students can focus on the concepts rather than making the diagrams. Colored pencils can be used for vertex coloring to aid in determining the least number of colors to be used.
Technology

Students can create a multimedia presentation to explore the Traveling Salesman Problem. Students are given a vertex-edge diagram of n cities as the vertices and the edges marked with the distance between each pair of cities. Calculate the number of distinct tours that are possible and the total distance that would be traveled in each case. Include a visual representation, such as a tree diagram, of the possible tours. Indicate which tour provides the minimum distance traveled. As an alternative, students can write a computer program that accepts as input the number of cities and produces as output the possible tours.

College and Workplace Readiness

Students can create a program scheduling public works employees at various job sites for one month. Graph Theory/Networking is used for scheduling in many careers such as operations management, mail delivery, and sales.