FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

SCIENCE AND ENGINEERING

HONORS ADVANCED STUDY MATHEMATICS

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 31, 2009

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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Course Philosophy

Students will learn to appreciate the value of Calculus as the mathematics of change and motion. In helping the students to achieve these goals, a variety of techniques are used to accommodate the different backgrounds, learning styles, and academic and career goals of the students. This course will prepare students for further study in all branches of higher mathematics, science and related fields. Calculus is not only the language for expressing physical laws in precise mathematical terms but it is also a tool for studying these laws. The course emphasizes an approach to Calculus that involves problems being expressed numerically, analytically, graphically and verbally. Technology and laboratory work are used as appropriate to reinforce these approaches.

Course Description

This course is an introduction to multivariable calculus. Topics include: calculus of functions of several variables, double and triple integrals, line integrals, vector fields, Green's theorem, Stokes' theorem, and the divergence theorem. Additionally, applications of these tools will be explored. Other topics include and introduction to linear algebra and matrix methods.

Freehold Regional High School District
Curriculum Map

S&E Honors Advanced Study in Mathematics

Relevant Standards [1]	Enduring Understandings	Essential Questions	Assessments		
			Diagnostic (before)	Formative (during)	Summative (after)
Unit 0 4.1A3, 4.2B1, 4.3A2, B2, C1, 2, 4.5B1-4, D1-6	Parametric systems are used to represent curves and surfaces. Limits can be used to analyze the behavior of functions of one or more variables.	Why are parametric systems often preferable to equation form for representing curves? How is the definition of a limit used to show a limit?	Summer Assignment Oral Questions/ Discussion Test on prerequisite material	Homework Quizzes Lab Assignments Chapter Test Written Assignments Oral Presentations Research Assignments	Projects Mid Terms Final Exam
Unit 1 4.2B1, 4.2C1, 4.2C2, 4.5A1-5, 4.5B1-4	Vectors are used to represent many physical characteristics. An appropriate coordinate system should be selected for each situation.	What is the difference between a vector and a scalar? How are vectors used to represent physical characteristics? How would you determine an appropriate coordinate system for a particular situation? How are vectors represented in noncartesian systems?			
Unit 2 4.2B1, C1, 2 4.3A2, B2, C1-2, 4.5A1-5, B1-4	Vector functions are useful in analyzing scalar systems and physical systems.	How do vector functions differ from parametric functions?			
Unit 3 4.1A3, 4.1C1, 4.3A2, 4.3B2, 4.3C1-2, 4.4A2, 4.4A4, 4.5A1-5, 4.5D1-6	Limits can be used to analyze the behavior of functions of one or more variables. There is a relationship between differential quantities and rates of change. Calculus of functions in several variables is the basis of many common analytical techniques that do not involve calculus.	How do limits of more than one variable differ from limits of one variable? How do limit properties lead to the differences between the differentials or derivatives of functions of one variable and functions of more than one variable? What physical interpretations can be applied to multivariable derivative?			

Relevant Standards [1]	Enduring Understandings	Essential Questions	Assessments		
			Diagnostic (before)	Formative (during)	Summative (after)
	There is a relationship between derivatives of functions of one variable to derivatives of functions of multiple variables, but they are not the same thing.				
Unit 4 4.2E2, 4.3A2, 4.3C1-2, 4.5A1-5, 4.5B1-4, 4.5C1-4	Integration is inherently a summing process in one dimension. Integrals can be iterated to evaluate systems involving more than one independent variable. Multiple integration is the general tool for problems involving volume and surface areas.	How is integration in multiple dimensions reduced to integration in one dimension? How are bounds interpreted and developed? How is multiple integration used to generalize solutions for previously studied applications? How can varying the order of integration simplify evaluation?			
Unit 5 4.2B1, 4.2C1, 4.2C2, 4.3A2, 4.3B2, 4.3C1-2, 4.5A1-5, 4.5B1-4, 4.5C1-4, 4.5D1-6	Vector fields and spaces are the basis of modern physics, chemistry, engineering and many other topical areas. There is a definite relationship between scalar functions and vector spaces.	What is the relation between vector fields and scalar spaces? How can the relationship between vector fields and scalar spaces be used to find solutions to particular problems? What physical interpretations can be applied to vector field concepts such as curl and divergence?			
Unit 6 4.1A3, 4.1B3, 4.2B1, 4.3C1-2, 4.5A1-5, 4.5 B1-4, 4.5C1-4, 4.5D1-6	There is a definite relationship between matrices and linear transformations.	What is the relationship between matrices and general linear transformations?			

Freehold Regional High School District
Course Proficiencies and Pacing

S&E Honors Advanced Study in Mathematics

Unit Title	Unit Understandings and Goals	Recommended Duration
Unit #0: Parametrics, Limits, and Trigonometric Substitution	Parametric systems are used to represent curves and surfaces. Limits can be used to analyze the behavior of functions of one or more variables. 1. The student will be able to analyze and use the properties of parametric systems of one variable. 2. The student will use advanced methods for finding limits and be able to use the definition of the limit. 3. The student will extend his/her understanding of integration techniques.	Summer work and 1 week
Unit #1: Vectors and the Geometry of Space	Vectors are used to represent many physical characteristics. An appropriate coordinate system should be selected for each situation. 1. The student will understand vector concepts in two or more dimensions. 2. The student will understand special properties of vectors in two or three dimensions. 3. The student will perform vector operations. 4. The student will understand relationships of vectors to lines, planes, and common surfaces. 5. The student will understand and use representations of vectors and vector concepts in various coordinate systems.	6 weeks
Unit #2: Vector-Valued Functions	Vector functions are useful in analyzing scalar systems and physical systems. 1. The student will understand representation and interpretation of vector valued functions. 2. The student will perform differentiation and integration on vector valued functions. 3. The student will use vector valued function to analyze applications. 4. The student will understand and use special vectors including tangent and normal. 5. The student will understand and use curvature and arc length.	6 weeks
Unit #3: Functions of Several Variables	Limits can be used to analyze the behavior of functions of one or more variables. There is a relationship between differential quantities and rates of change. Calculus of functions in several variables is the basis of many common analytical techniques that do not involve calculus. There is a relationship between derivatives of functions of one variable to derivatives of functions of multiple variables, but they are not the same thing. 1. The student will understand how to write and interpret a function of more than one variable. 2. The student will understand how to evaluate the limit of a function of more than one variable. 3. The student will use the definition of a limit to demonstrate the limit values. 4. The student will find and use first, second, etc. orders of partial derivatives. 5. The student will use partial derivatives to find tangent planes, normal lines, gradients, etc. 6. The student will use partial derivatives and other techniques to find extrema and other special points.	6 weeks

Unit Title	Unit Understandings and Goals	Recommended Duration
Unit #4: Multiple Integration	Integration is inherently a summing process in one dimension. Integrals can be iterated to evaluate systems involving more than one independent variable. Multiple integration is the general tool for problems involving volume and surface areas. 1. The student will understand and use the syntax and methods of evaluation for iterated integrals. 2. The student will understand and use multiple integration in a variety of applications such as volume, mass, and surface area. 3. The student will understand and demonstrate integration in various coordinate systems and transformations between systems.	6 weeks
Unit #5: Vector Analysis	Vector fields and spaces are the basis of modern physics, chemistry, engineering and many other topical areas. There is a definite relationship between scalar functions and vector spaces. 1. The student will understand representations of vector fields. 2. The student will write functions describing vector fields. 3. The student will analyze applications using vector fields. 4. The student will understand and use line integrals in scalar spaces and vector fields. 5. The student will understand and use surface integrals in scalar spaces and vector fields. 6. The student will understand and use important theorems and properties that relate scalar spaces and vector fields	6 weeks
Unit #6: Additional Topics	There is a definite relationship between matrices and linear .transformations. 1. The student will understand elementary matrix operations. 2. The student will examine elementary applications of matrices. 3. Students will understand that matrices represent general special transformations. 4. Students will examine relationships between multivariable calculus tools and transformations using matrices.	Up to 4 weeks

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S&E Honors Advanced Study in Mathematics

Unit #0: Parametrics, Limits, and Trigonometric Substitution

Enduring Understandings: Parametric systems are used to represent curves and surfaces.
Limits can be used to analyze the behavior of functions of one or more variables.
Essential Questions: Why are parametric systems often preferable to equation form for representing curves?
How is the definition of a limit used to show a limit?
Unit Goals: The student will be able to analyze and use the properties of parametric systems of one variable.
The student will use advanced methods for finding limits and be able to use the definition of the limit.
The student will extend his/her understanding of integration techniques.
Duration of Unit: Summer and 1 week
NJCCCS: 4.1A3, 4.2B1, 4.3A2, 4.3B2, 4.3C1, 4.3C2, 4.5B1-4, 4.5D1-6

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
How do we write parametric systems? How do the system components relate to the independent parameter? What characteristics differentiate curve defined by parametric systems from curves defined by simple functions? How can we find slope, arch length and other characteristics of a curve? How are limits defined? How do we know that a limit is correct? What quadratic surfaces are most common? What are the most common alternate coordinate system representations?	Define syntax for parametric systems. Define relations between component functions of a system. Relate parametric curve characteristics to characteristics of simple functions. Define integration and differentiation of parametric systems. Use the formal definition of a limit. Prove the correctness of limit values using the definition of limits. Examine standard surfaces such as hyperboloid, ellipsoid, and paraboloid. Define and work with alternate coordinate systems such as spherical and cylindrical.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

Suggestions on how to differentiate in this unit:

- The use of cooperative learning, alternate assessment that includes projects and presentations, along with the use of technology, will meet the needs of any diverse learner.

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Unit #1: Vectors and the Geometry of Space

Enduring Understandings: Vectors are used to represent many physical characteristics.
An appropriate coordinate system should be selected for each situation.

Essential Questions: What is the difference between a vector and a scalar?
How are vectors used to represent physical characteristics?
How would you determine an appropriate coordinate system for a particular situation?
How are vectors represented in noncartesian systems?

Unit Goals: The student will understand vector concepts in two or more dimensions.
The student will understand special properties of vectors in two or three dimensions.
The student will perform vector operations.
The student will understand relationships of vectors to lines, planes, and common surfaces.
The student will understand and use representations of vectors and vector concepts in various coordinate systems.

Duration of Unit: 6 weeks

NJCCCS: 4.2B1, 4.2C1, 4.2C2, 4.5A1-5, 4.5B1-4

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
How are vectors expressed? How can we manipulate vectors? What special vectors do we commonly use? How can vectors be used to specify a particular line? How can vectors be used to specify a particular plane? What other common operations are performed with vectors? How are vectors used to solve problems?	Define component and direction magnitude form for vectors. Define elementary properties and operations on vectors, such as sum, difference, scalar multiple, magnitude, dot product and cross product. Identify standard vectors, such as coordinate axis vectors and unit vectors. Develop relationship between vectors and equations for lines. Develop relationships between vectors and planes. Examine advanced operations such as projection. Examine and solve application problems.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

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- The use of cooperative learning, alternate assessment that includes projects and presentations, along with the use of technology, will meet the needs of any diverse learner.

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Unit #2: Vector-Valued Functions

Enduring Understanding: Vector functions are useful in analyzing scalar systems and physical systems.
Essential Question: How do vector functions differ from parametrics?
Unit Goals: The student will understand representation and interpretation of vector valued functions.
The student will perform differentiation and integration on vector valued functions.
The student will use vector valued function to analyze applications.
The student will understand and use special vectors including tangent and normal.
The student will understand and use curvature and arc length.
Duration of Unit: 6 weeks
NJCCCS: 4.2B1, 4.2C1, 4.2C2, 4.3A2, 4.3B2, 4.3C1-2, 4.5A1-5, 4.5B1-4

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
How are vector-valued functions written? What is the most common representation of a vector-valued function? How do limit concepts extend to vector-valued functions? How do we find, represent and interpret derivatives of vector-valued functions? How do we find, represent, and interpret integrals of vector-valued functions? Where are vector-valued functions used? What special vectors come with vector valued functions? How can we use operations on vector-valued functions and the special vectors to derive additional tools?	Vector valued functions are built from parametric systems. Define graphical representation of curves described by vector-valued functions. Define and use limits of vector-valued functions. Define and use derivatives of vector-valued functions and define and use the properties of these derivatives. Define and use integration of vector-valued functions and define and use the properties of these integrals Examine applications of derivatives and integrals of vector-valued functions. Define and examine special vectors including unit, tangent, normal, and binormal. Derive and use arch length and curvature tools.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

<u>Suggestions on how to differentiate in this unit</u>:

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Unit #3: Functions of Several Variables

Enduring Understandings: Limits can be used to analyze the behavior of functions of one or more variables.
There is a relationship between differential quantities and rates of change.
Calculus of functions in several variables is the basis of many common analytical techniques that do not involve calculus.
There is a relationship between derivatives of functions of one variable to derivatives of functions of multiple variables, but they are not the same thing.

Essential Questions: How do limits of more than one variable differ from limits of one variable?
How do limit properties lead to the differences between the differentials or derivatives of functions of one variable and functions of more than one variable?
What physical interpretations can be applied to multivariable derivative?

Unit Goals: The student will understand how to write and interpret a function of more than one variable.
The student will understand how to evaluate the limit of a function of more than one variable.
The student will use the definition of a limit to demonstrate the limit values.
The student will find and use first, second, etc. orders of partial derivatives.
The student will use partial derivatives to find tangent planes, normal lines, gradients, etc.
The student will use partial derivatives and other techniques to find extrema and other special points.

Duration of Unit: 6 weeks

NJCCCS: 4.1A3, 4.1C1, 4.3A2, 4.3B2, 4.3C1-2, 4.4A2, 4.4A4, 4.5A1-5, 4.5D1-6

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
What does a function of more than one variable represent? And what are useful tools in interpreting functions of more than one variable? What basic tools of differential calculus apply to functions of more than one variable? How do the basic tools of differential calculus differ when applied to a function of a single variable and a function of more than one variable? What new derivative concepts apply to a function of more than one variable? How does direction in our domain relate to rate of change? How do we find tangent and normal lines to curves or surfaces? How do we find critical points? What are the common applications of critical points?	Define and use level curves, level surfaces, and traces. Define and use the limit of a function of more than one variable. Explore path dependence as it relates to limits. Define and use continuity concepts and partial derivatives of nth order. Define and use differentials including differential approximation and differential error concepts. Explore the physical interpretation of a function of more than one variable. Develop and explore the differences between the derivative of a function of one variable and the derivative of a function of more than one variable. Use the chain rule to develop implicit differentiation and solve problems. Develop and use directional derivatives and gradient and explore the relationship between them. Use derivatives and gradients to find tangents and normals to curves and surfaces. Use partial derivatives to identify and classify critical points and extrema and to develop tools such as least squares regression. Explore and use the method of Lagrange multipliers as an application of gradient.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

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Unit #4: Multiple Integration

Enduring Understandings: Integration is inherently a summing process in one dimension.
Integrals can be iterated to evaluate systems involving more than one independent variable.
Multiple integration is the general tool for problems involving volume and surface areas

Essential Questions: How is integration in multiple dimensions reduced to integration in one dimension?
How are bounds interpreted and developed?
How is multiple integration used to generalize solutions for previously studied applications?
How can varying the order of integration simplify evaluation?

Unit Goals: The student will understand and use the syntax and methods of evaluation for iterated integrals.
The student will understand and use multiple integration in a variety of applications such as volume, mass, and surface area.
The student will understand and demonstrate integration in various coordinate systems and transformations between systems.

Duration of Unit: 6 weeks

NJCCCS: 4.2E2, 4.3A2, 4.3C1-2, 4.5A1-5, 4.5B1-4, 4.5C1-4

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
What is multiple integration? How are regions in the domain expressed?	Define iterated integration. Examine how to write limits of integration. Examine how to change order of integration. Apply iterated integration to find area and volume. Develop tools for multiple integration in domains represented in alternate coordinate systems. Explore applications of multiple integration in physics. Use multiple integration to find surface area. Derive change of variable techniques using the Jacobian matrix.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

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Unit #5: Vector Analysis

Enduring Understandings: Vector fields and spaces are the basis of modern physics, chemistry, engineering and many other topical areas.
There is a definite relationship between scalar functions and vector spaces.

Essential Questions: What is the relation between vector fields and scalar spaces?
How can the relationship between vector fields and scalar spaces be used to find solutions to particular problems?
What physical interpretations can be applied to vector field concepts such as curl and divergence?

Unit Goals: The student will understand representations of vector fields.
The student will write functions describing vector fields.
The student will analyze applications using vector fields.
The student will understand and use line integrals in scalar spaces and vector fields.
The student will understand and use surface integrals in scalar spaces and vector fields.
The student will understand and use important theorems and properties that relate scalar spaces and vector fields

Duration of Unit: 6 weeks

NJCCCS: 4.2B1, 4.2C1, 4.2C2, 4.3A2, 4.3B2, 4.3C1-2, 4.5A1-5, 4.5B1-4, 4.5C1-4, 4.5D1-6

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
What are common types of vector fields? How are vector fields related to scalar functions? What are some common operations on vector fields? How can we integrate a function along a path? When do we have special paths and what are they? How can we integrate a function over a surface? What special surfaces do we have? What happens when you try to perform a surface integral on a nonorientable surface?	Examine specific common vector fields such as inverse square central field. Define conservative field and explore the relationship of a conservative field to the gradient of a scalar function. Define and explore applications of divergence. Explore the physical interpretations of divergence and curl. Define and use line integrals in scalar space and vector fields. Define and use the Fundamental Theorem of Line Integrals. Use the concept of path independence in a conservative field. Use Green's Theorem. Explore and use parametric surfaces including applications such as finding surface area and surface integrals. Explore and use flux integrals and use the Divergence Theorem and Stokes's Theorem.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

Suggestions on how to differentiate in this unit:

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Freehold Regional High School District
S&E Honors Advanced Study in Mathematics

Unit #6: Additional Topics

Enduring Understanding: There is a definite relationship between matrices and linear transformations.
Essential Question: What is the relationship between matrices and general linear transformations?
Unit Goals: The student will understand elementary matrix operations.
The student will examine elementary applications of matrices.
Students will understand that matrices represent general special transformations.
Students will examine relationships between multivariable calculus tools and transformations using matrices.
Duration of Unit: Up to 4 weeks
NJCCCS: 4.1A3, 4.1B3, 4.2B1, 4.3C1-2, 4.5A1-5, 4.5 B1-4, 4.5C1-4, 4.5D1-6

Guiding / Topical Questions	Content, Themes, Concepts, and Skills	Instructional Resources and Materials	Teaching Strategies	Assessment Strategies
How do we write matrices? How do operate on matrices? What are elementary applications of matrices? What are special properties of the elementary operations of matrices? How do matrix operations relate to topics in multivariable calculus?	Define matrix. Define and use matrix addition, multiplication, and inversion. Use matrices to solve systems of equations. Examine the relationship between matrix inversion and solving a system of equations. Explore limit properties of matrix products. Explore matrix products as general coordinate system transformation.	Textbook Hand outs Internet resources Electronic tools	Lecture and class discussion Sample problems Guided practice Development of physical models	Written tests and quizzes Worksheets Project assessments Responses to discussion questions

Suggestions on how to differentiate in this unit:

- The use of cooperative learning, alternate assessment that includes projects and presentations, along with the use of technology, will meet the needs of any diverse learner.