

**FREEHOLD REGIONAL HIGH SCHOOL DISTRICT**

**OFFICE OF CURRICULUM AND INSTRUCTION**

**SCIENCE AND ENGINEERING**

# **ADVANCED PLACEMENT CALCULUS BC**

Grade Level: 11

Credits: 5

**BOARD OF EDUCATION ADOPTION DATE:**

**AUGUST 25, 2008**

[SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING](#)

APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS

APPENDIX B: ASSESSMENT EVIDENCE

APPENDIX C: INTERDISCIPLINARY CONNECTIONS

# **FREEHOLD REGIONAL HIGH SCHOOL DISTRICT**

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## **COURSE SPECIFICATIONS**

Duration: One year  
Credits: 10 credits---Honors  
Prerequisites: Science/Engineering mathematics II

## **COURSE DESCRIPTION**

Science / Engineering Calculus is a full year, ten credit course in calculus. It is designed for students who plan on majoring in one of the specialized areas of engineering, science or mathematics. Calculus is explored through the interpretation of graphs and tables as well as analytic methods (multiple representation of functions). Derivatives are interpreted as rates of change and local linear approximation. The definite integral is interpreted as total change over a specific interval and as a limit Riemann sum. The uses of technology is integrated throughout the book to provide a balanced approach to the teaching and learning of calculus that involves algebraic, numerical, graphical, verbal, and written methods. Rich array of interesting applications in biology, business, chemistry, engineering, physics, the social sciences, and statistics are included in the course of learning. Analytical and critical thinking will be stressed at all times as research indicates that mastery of the skills required for success in the Science / Engineering fields.





## COURSE PROFICIENCIES

### Science / Engineering Calculus BC

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Students enrolled in this course will demonstrate mastery of the following proficiency requirements as outlined in the curriculum guide and receive a passing grade in accordance with Board of Education policies on grading and attendance:

#### 1. Functions, Graphs, and limits

- With the aid of technology, students should be able to produce the graphs of functions. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.
- Students are to have an intuitive understanding of the limiting process, be able to calculate limits using algebra and estimating limits from graphs or tables of data
- Students are to understand asymptotes in terms of graphical behavior, describe asymptotic behavior in terms of limits involving infinity, and compare relative magnitudes of functions and their rates of change.
- Students should have the central idea of continuity, understand continuity in terms of limits, and have geometric understanding of graphs of continuous functions.
- Students are also encouraged to analyze the planar curves including those given in parametric form, polar form, and vector form.

#### 2. DERIVATIVES

- Students must understand the concept of derivative is presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change.
- Students should understand the relationship between the slope of a curve at a point and tangent line to a curve at a point; grasp the method of local linear approximation.
- Students are to learn the corresponding characteristics of the graphs of  $f$ ,  $f'$  and  $f''$ , relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$ , the Mean Value Theorem and its geometric consequence, relationship between the concavity of  $f$  and sign of  $f''$ , and points of inflection as places where concavity changes.
- Students are to use the application of derivatives to analyze curves, optimization, and modeling; find the derivative of an inverse function by use of implicit differentiation and interpretation of the derivative as a rate of change in varied applied contexts, geometric interpretation of differential equations, numerical solution of differential equations using Euler's method, and L'Hopital's Rule and its use in determining convergence of improper integrals and series.
- Students must have the knowledge of derivatives of basic functions basic rules for the derivatives, chain rule and implicit differentiation, and derivatives of parametric, polar and vector functions.

### 3. INTEGRALS

- Students must have a clear understanding of Riemann sum and be able to compute Riemann sums using left, right and midpoint evaluation points
- Students must know the interpretations and properties of definite integrals
- Students must also know the applications of appropriate integrals used in a variety of situations.
- Students are to use the Fundamental Theorem of Calculus to evaluate definite integrals and to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.
- Students should also learn the techniques of antidifferentiation and applications of antidifferentiation

### 4. POLYNOMIAL APPROXIMATIONS AND SERIES

- Students are to learn the concept of series and use technology to explore convergence or divergence of various examples.
- With series of constants, students are to grasp the skills regarding decimal expansion, geometric series with applications, the harmonic series, alternating series with error bound, terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series, the ratio test for convergence and divergence, and comparing series to test for convergence or divergence.
- Students are expected to do Taylor polynomial approximation with graphical demonstration of convergence. Formal manipulation of Taylor series and shortcuts to compute Taylor series, including differentiation, antidifferentiation, and the formation of new series from known series are required.

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Somewhere in a New Jersey elementary school

Somewhere in a New Jersey high school

Standard 4.1: All Students Will Develop The Ability To Pose And Solve Mathematical Problems In Mathematics, Other Disciplines, And Everyday Experiences.

Standard 4.2: All Students Will Communicate Mathematically Through Written, Oral, Symbolic, And Visual Forms Of Expression.

Standard 4.3: All Students Will Connect Mathematics To Other Learning By Understanding The Interrelationships Of Mathematical Ideas And The Roles That Mathematics And Mathematical Modeling Play In Other Disciplines And In Life.

Standard 4.4: All Students Will Develop Reasoning Ability And Will Become Self-Reliant, Independent Mathematical Thinkers.

Standard 4.5: All Students Will Regularly And Routinely Use Calculators, Computers, Manipulatives, And Other Mathematical Tools To Enhance Mathematical Thinking, Understanding, And Power.

Standard 4.6: All Students Will Develop Number Sense And An Ability To Represent Numbers In A Variety Of Forms And Use Numbers In Diverse Situations.

Standard 4.7: All Students Will Develop Spatial Sense And An Ability To Use Geometric Properties And Relationships To Solve Problems In Mathematics And In Everyday Life.

Standard 4.8: All Students Will Understand, Select, And Apply Various Methods Of Performing Numerical Operations.

Standard 4.9: All Students Will Develop An Understanding Of And Will Use Measurement To Describe And Analyze Phenomena.

Education, New Jersey State Curriculum Content Standards

<http://www.state.nj.us/edec>

Standard 4.10: All Students Will Use A Variety Of Estimation Strategies And Recognize Situations In Which Estimation Is Appropriate.

Standard 4.11: All Students Will Develop An Understanding Of Patterns, Relationships, And Functions And Will Use Them To Represent And Explain Real-World Phenomena.

Standard 4.12: All Students Will Develop An Understanding Of Statistics And Probability And Will Use Them To Describe Sets Of Data, Model Situations, And Support Appropriate Inferences And Arguments.

Standard 4.13: All Students Will Develop An Understanding Of Algebraic Concepts And Processes And Will Use Them To Represent And Analyze Relationships Among Variable Quantities And To Solve Problems.

Standard 4.14: All Students Will Apply The Concepts And Methods Of Discrete Mathematics To Model And Explore A Variety Of Practical Situations.

Standard 4.15: All Students Will Develop An Understanding Of The Conceptual Building Blocks Of Calculus And Will Use Them To Model And Analyze Natural Phenomena.

Standard 4.16: All Students Will Demonstrate High Levels Of Mathematical Thought Through Experiences Which Extend Beyond Traditional Computation, Algebra, And Geometry.

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## INSTRUCTIONAL OBJECTIVES

- Students should understand the meaning of the derivative in terms of rate of change and local linear approximations.
- Students should be able to work with functions represented graphically, numerically, analytically, or verbally, and should understand the connections among these representations.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as a net accumulation of a rate of change.
- Students should be able to model problem situations with functions, differential equations, or integrals.
- Students should be able to represent differential equations with slope fields, solve separable differential equations analytically, and solve differential equations using numerical techniques such as Euler's method.
- Students should be able to interpret convergence and divergence of series using technology. They should be able to represent functions with series and find the Lagrange error bound for Taylor polynomials.
- Students are expected to use a multirepresentational approach to investigate and solve problems, to write about their conclusions, and often to work in groups to communicate mathematics orally.
- Students must value all of the methods of representation including: analytic, algebraic, numerical, graphical, and verbal methods, and understand how they are connected in a given problem and learn how to choose the one(s) most appropriate for solving a particular problem.
- Students are asked to solve some problems by one method and then support or confirm their solutions by using another method. Students must understand that mathematics provides the foundation that allows us to use technology to solve problems.
- Students are exposed to functions as mechanisms for modeling data and learn about how various functions can model real-life problems. They learn to analyze and model data, represent data graphically, interpret from graphs and fit curves.
- Students are expected to be actively involved in understanding calculus concepts and solving problems. The explorations help build problem-solving ability by building students to develop a mathematical model of a problem, solve the mathematical model, support or confirm the solution, and interpret the solution.

## COURSE CONTENT AND SUGGESTED TIME LINE

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<b>Chapter 1 Prerequisites for Calculus</b>	<b>4 days</b>
1.1 Lines	0
1.2 Functions and Graphs	0
1.3 Exponential Functions	1
1.4 Parametric Equations	1
1.5 Functions and Logarithms	1
1.6 Trigonometric Functions	1
Review Exercises/Test	
<b>Chapter 2 Limits and Continuity</b>	<b>6 days</b>
2.1 Rates of Change and Limits	1
2.2 Limits Involving Infinity	1
2.3 Continuity	1
2.4 Rates of Change and Tangent Lines	1
Review Exercises/Test	2
<b>Chapter 3 Derivatives</b>	<b>22 days</b>
3.1 Derivative of a Function	2
3.2 Differentiability	2
3.3 Rules for Differentiation	2
3.4 Velocity and Other Rates of Change	2
3.5 Derivatives of Trigonometric Functions	2
3.6 Chain Rule	2
3.7 Implicit Differentiation	2
3.8 Derivatives of Inverse Trigonometric Functions	2
3.9 Derivatives of Exponential and Logarithmic Functions	3
Review Exercises/Test	3
<b>Chapter 4 Applications of Derivatives</b>	<b>14 days</b>
4.1 Extreme Values of Functions	2
4.2 Mean Value Theorem	1
4.3 Connecting $f'$ and $f''$ with the Graph of $f$	2
4.4 Modeling and Optimization	3
4.5 Linearization (and Newton's Method optional)	1
4.6 Related Rates	3
Review Exercises/Test	2
<b>Chapter 5 The Definite Integral</b>	<b>16 days</b>
5.1 Estimating with Finite Sums	2
5.2 Definite Integrals	3
5.3 Definite Integrals and Antiderivatives	5
5.4 Fundamental Theorem of Calculus	3
5.5 Trapezoidal Rule	1
Review Exercises/Test	2

<b>Chapter 6 Differential Equations and Mathematical Modeling</b>	<b>22 days</b>
6.1 Antiderivatives and Slope Fields	3
6.2 Integration by Substitution	3
6.3 Integration by Parts	2
6.4 Exponential Growth and Decay	3
6.5 Population Growth	5
6.6 Numerical Methods	3
Review Exercises/Test	3

<b>Chapter 7 Applications of Definite Integrals</b>	<b>15 days</b>
7.1 Integral as Net Change	4
7.2 Areas in the Plane	2
7.3 Volumes	3
7.4 Lengths of Curves	2
7.5 Physical Applications	2
Review Exercises/Test	3

<b>Chapter 8 L'Hôpital's Rule, Improper Integrals, and Partial Fractions</b>	<b>7 days</b>
8.1 L'Hôpital's Rule	1
8.2 Relative Rates of Growth	1
8.3 Improper Integrals	2
8.4 Partial Fractions and Integral Tables	1
Review Exercises/Test	2

<b>Chapter 9 Infinite Series</b>	<b>23 days</b>
9.1 Power Series	4
9.2 Taylor Series	4
9.3 Taylor's Theorem	4
9.4 Radius of Convergence	3
9.5 Testing Convergence at Endpoints	5
Review Exercises/Test	3

<b>Chapter 10 Parametric, Vector, and Polar Functions</b>	<b>16 days</b>
10.1 Parametric Equations	2
10.2 Vectors in Plane	2
10.3 Vector-valued Functions	3
10.4 Modeling Projectile Motion	2
10.5 Polar Coordinates and Polar Graphs	2
10.6 Calculus of Polar Curves	3
Review Exercises/Test	2

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**NOTE:** If Chapter 1 and Chapter 2 were covered in the Pre-calculus course the previous year, then, with approximately 150 teaching days before the AP Exam, the students will have approximately 25 days to review the course.

## INSTRUCTIONAL METHODOLOGY

The following instructional methods will be employed by the teacher of the Science/Engineering Calculus course. This is, of course, a menu of possible choices available to the teacher who will bring his or her own creativity and talents to the classroom to augment the suggestions given here.

1. Group lecture
2. Questioning
3. Small/large group discussion
4. Oral and written presentations by the students
5. Guest lecturers
6. Field trips
7. Related computer activities
8. Independent study

## METHODS OF EVALUATION

Grading in the Science/Engineering calculus course will follow the guidelines established by the Freehold Regional Board of Education. In any educational setting, proof of mastery of required material is a necessary part of the system. Methods used to evaluate that mastery may include, but not be limited to the following:

1. Teacher unit tests
2. Standardized tests
3. Quizzes
4. Classroom discussion
5. Oral reports
6. Written reports
7. Computer programs
8. Homework
9. Interdisciplinary work
10. Midterm and final examination