FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

MATHEMATICS DEPARTMENT

HONORS & ACADEMIC PRE-CALCULUS

Grade Level: 10-12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 29, 2016

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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PRE-CALCULUS
COURSE PHILOSOPHY
Pre-Calculus provides a balance of analytical techniques and theoretical instruction. This course explores recurring themes across mathematical domains and strengthens mathematical proficiency through problem solving, inquiry, and discovery. *Pre-Calculus* emphasizes modeling, reasoning, and communicating mathematically through the use of technology. *Pre-Calculus* provides students with a strong foundation for success in the study of calculus.
COURSE DESCRIPTION
Pre-Calculus is the study of discrete topics in advanced algebra and trigonometry. Students will investigate theoretical, numerical, graphical, and spatial topics upon which to build their study of advanced mathematics. *Pre-Calculus* provides the background for mathematical concepts, problems, issues, and techniques that appear in the study of calculus, including but not limited to: functions, trigonometry, polynomials, complex numbers, matrices, series and sequences, limits and continuity, and derivatives. The use of technology is infused in this course to gather, analyze, and communicate mathematical information.

COURSE SUMMARY	
COURSE GOALS	
Students will investigate and analyze relations and functions in order to construct graphic, algebraic, and linguistic models of real-world situations.	
COURSE ENDURING UNDERSTANDINGS	COURSE ESSENTIAL QUESTIONS
CEU1: Functions and their properties can be used to model and analyze real-world situations to solve problems and make predictions.	CEQ1a: Can all relationships in the real-world be modeled with functions? CEQ1b: How can I use functions to predict real-world events? CEQ1c: How do I know which function will best model the scenario?
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?

UNIT GOALS & PACING		
UNIT TITLE	**UNIT GOALS**	**RECOMMENDED DURATION**
Unit 1: Functions	Students will create graphical and algebraic models of functions and their transformations and interpret key properties of the functions.	2-3 weeks
Unit 2: Trigonometry	Students will extend the domain of trigonometric functions using the unit circle, model periodic phenomena using trigonometric functions, and prove and apply trigonometric identities.	10-12 weeks
Unit 3: Matrices (Optional)	Students will use matrices and technology as a tool to manipulate data and model equations.	1-2 weeks
Unit 4: Vector and Parametric Equations (Optional)	Students will use vectors and parametric equations to model movement in the coordinate plane and the physical world.	1-2 weeks
Unit 5: Polynomial and Rational Functions	Students will graph and analyze polynomial and rational functions in order to predict end behavior.	3-4 weeks
Unit 6: Exponentials and Logarithms	Students will model, interpret, and make predictions about exponential and logarithmic relationships.	3 weeks
Unit 7: Series & Sequences	Students will analyze and create sequences and series that model the long-term behavior of situations involving sequential, arithmetic, or geometric change.	2 weeks
Unit 8: Limits, Continuity and Basic Derivatives (Optional - Applications of Derivatives)	Students will use limits to describe the instantaneous rate of change of functions and predict behavior.	6-7 weeks
Unit 9: Conics (Optional)	Students will connect conic sections to their quadratic forms in order to create graphical models.	0-2 weeks

PRE-CALCULUS
UNIT 1: Functions **SUGGESTED DURATION: 2-3 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will create graphical and algebraic models of functions and their transformations and interpret key properties of the functions.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, students will create, solve, and explain an application scenario that coincides with a function.
3	The student can: • transform functions on a graph; • describe transformations and properties of functions; • identify the domain and range of a function and its inverse; and • determine the composition of two functions and its domain.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU1: Functions and their properties can be used to model and analyze real-world situations to solve problems and make predictions.	CEQ1a: Can all relationships in the real-world be modeled with functions? CEQ1b: How can I use functions to predict real-world events? CEQ1c: How do I know which function will best model the scenario?
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: A parent function can provide insight into the function's behavior.	EQ1: Why do we need to model functions in various ways?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 CEU1, CEQ1a, b, c CEU2, CEQ2a, b, c EU1, EQ1 F.BF.A.1c F.BF.B.3, 4a, 4b, 4c, 4d F.IF.B.4, 5 F.IF.C.7b DOK 2	Students will analyze a graph of a function to determine its parent function. They will justify their answer mathematically by describing domain, range, maximums, minimums, zeros, intercepts, and symmetry of the function.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
arithmetic functions composition of functions constant functions decreasing intervals domain function notation horizontal test increasing intervals intercepts inverse functions odd and even functions one-to-one line test piecewise functions range relative maxima and minima slope-intercept transformations vertical line test	Compose functions that represent real-world scenarios (DOK 2)	F.BF.A.1c Write a function that describes a relationship between two quantities: compose functions.
	Graph the transformation of a given function for specific values (both positive and negative) (DOK 2)	F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
	Solve an equation of the form f(x)= c for a simple function f that has an inverse and write an expression for the inverse (DOK 2)	F.BF.B.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
	Verify by composition that one function is the inverse of another (DOK 2)	F.BF.B.4b Verify by composition that one function is the inverse of another.
	Connect the properties of inverse functions through the analysis of a table or graph (DOK 3)	F.BF.B.4c Read values of an inverse function from a graph or a table, given that the function has an inverse.
	Produce an invertible function from a non-invertible function by restricting the domain (DOK 3)	F.BF.B.4d Produce an invertible function from a non-invertible function by restricting the domain.
	Sketch graphs showing the key features of a function and modeling a relationship between two quantities, given a verbal description of the relationship (DOK 2)	F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
	Relate the domain of a function to its graph and to the quantitative relationship it describes, where applicable (DOK 2) Develop a logical argument as to why a domain is appropriate for a given solution (DOK 3)	F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
	Graph square root, cube root, and piecewise-defined functions, including step-functions and absolute functions, by hand or by using technology, and show/label key features of the graph (DOK 2)	F.IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

PRE-CALCULUS
UNIT 2: Trigonometry **SUGGESTED DURATION: 10-12 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will extend the domain of trigonometric functions using the unit circle, model periodic phenomena using trigonometric functions, and prove and apply trigonometric identities.	
UNIT LEARNING SCALE	
4	In addition to level 3 performances, the student can create a real-world scenario that represents periodic phenomena.
3	The student can: • graph and analyze trigonometric functions; • connect the coordinates of the unit circle to the values of the trigonometric functions; • use inverse functions to solve trigonometric equations; • simplify, verify, and solve problems using trigonometric identities; and • derive and apply various trigonometric area formulas.
2	The student can • graph the parent trigonometric functions; • find exact values of a trigonometric function with the assistance of the unit circle; • solve basic trigonometric equations; • verify basic trigonometric identities; and • with assistance complete more complex tasks.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: The characteristics of trigonometric functions and their representations are useful in solving real-world problems.	EQ1: What relationships in the real-world can be modeled using trigonometry?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 CEU2, CEQ2a, b, c EU1, EQ1 F.IF.C.7e F.TF.A.1,2, 3, 4 F.TF.B.5, 6, 7 F.TF.C.8, 9 G.SRT.D.9, 10, 11 DOK 3	Students will complete each independent assessment: 1. Students will graph, solve, and evaluate trigonometric functions with and without technology. 2. Students will use the law of sines and/or the law of cosines to find the height of a building given two angles of elevation. 3. Students will use a sinusoidal model to predict the height of a person riding a Ferris wheel at a given time. 4. Students will prove trigonometric identities.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
amplitude angle measure angle of depression angles of elevation angular displacement	Graph trigonometric functions, by hand or by using technology, and show period, midline, and amplitude (DOK 2) Analyze the graph of a trigonometric function and determine if the graph is correct (DOK 3)	F.IF.C.7e Graph ~~exponential and logarithmic functions, showing intercepts and end behavior~~, and trigonometric functions, showing period, midline, and amplitude.
angular velocity asymptotes complex conjugates complex numbers complex plane cosecant cosine cotangent degrees	Apply angular velocity and linear velocity to real-world phenomena (DOK 3) Compare and contrast angular velocity and linear velocity (DOK 3) Define a radian measure of an angle as the length of the arc on the unit circle subtended by the angle (DOK 1) Define terminal and initial side of an angle on the unit circle (DOK 1)	F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
domain double angle identities half-angle identities imaginary numbers inverse functions linear velocity midline operations of complex numbers period periodic functions phase shift polar coordinates	Interpret and use the properties of all six trigonometric functions to identify trigonometric ratios, solve problems, and calculate exact values from the unit circle (sine, cosine, tangent, cosecant, secant, cotangent) (DOK 2) Explain the relationship between a counterclockwise radian measure of an angle along the unit circle, terminal coordinate on the unit circle of that angle, and the associated real number (DOK 2) Formulate how the radian measures of angles of the unit circle in the coordinate plane enable the extension of trigonometric functions to all real numbers (DOK 3)	F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
(continued from previous page) polar equations polar graphs • lemniscate • limaçon • rose • spiral polar/trigonometric form powers & roots of complex numbers properties of inverse functions	Determine the values of sine, cosine, and tangent, using special right triangles and apply their properties to real-world phenomena (DOK 2) Construct the values of sine, cosine, and tangent, using the unit circle, for any real number (DOK 3)	F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.
	Use the unit circle to explain symmetry of trigonometric functions (DOK 2) Use the unit circle to predict and identify patterns in the periodicity of trigonometric functions (DOK 2)	F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
properties of trigonometric functions Pythagorean identities quotient identities radians range reciprocal identities reference angle secant sine special right triangles sum and difference identities tangent transformation of functions transformation of polar graphs trigonometric equations trigonometric functions trigonometric identities trigonometric ratios unit circle vertical shift	Identify patterns and explain why real-world or mathematical phenomena exhibit characteristics of periodicity (DOK 2) Graph the trigonometric functions (D0K 2) Relate properties of graphical representations (DOK 2)	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
	Identify the domain and range of inverse trigonometric functions (DOK 1) Prove that restricting y = sin x, y = cos x, and y = tan x to a domain on which it is always increasing or decreasing allows its inverse to be constructed (DOK 3) Assess the phenomena of graphing inverse trigonometric functions in terms of the given trigonometric function (DOK 3)	F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
	Use inverse functions to solve trigonometric equations and analyze the solutions using technology to explain phenomena in non-routine problems (DOK 3)	F.TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
	Construct logical trigonometric arguments by manipulating trigonometric identities to show Pythagorean identities are true for all values on the unit circle (DOK 3) Prove the Pythagorean identities (DOK 3) Relate the Pythagorean identity to find sin(θ), cos(θ), or tan(θ), given sin(θ), cos(θ), or tan(θ), and the quadrant of the angle (DOK 2)	F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
	Use identity formulas for sine, cosine, and tangent to solve problems (DOK 2)	F.TF.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
	Derive and apply various trigonometric area formulas (DOK 3)	G.SRT.9 Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	Use the laws of sines and cosines to find missing angles or side length measurements (DOK 2)	G.SRT.10 Prove the laws of sines and cosines and use them to solve problems.
	Use the law of sines and law of cosines to solve problems involving non-right triangles that represent real-world phenomena (DOK2) Differentiate from given measurements in right and non-right triangles whether it is appropriate to use the law of sines or cosines (DOK 3) Apply the law of sines and the law of cosines to find unknown measurements in right and non-right triangles (DOK 2)	G.SRT.11 Understand and apply the law of sines and the law of cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
	Explain that the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers (DOK 2)	N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
	Identify patterns in conjugates to find moduli of complex numbers (DOK 2) Use conjugates to find quotients of complex numbers (DOK 2)	N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
	Represent complex numbers on the complex plane in rectangular form (DOK 1)	N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
	Use concepts of non-routine problems, real-world, or mathematical phenomena to exhibit characteristics of periodicity (DOK 3)	N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
end behavior identify transformation of functions	Sketch graphs showing the key features of a function and modeling a relationship between two quantities, given a verbal description of the relationship (DOK 2)	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

PRE-CALCULUS
UNIT 3: Matrices (Optional) **SUGGESTED DURATION: 1- 2 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will use matrices and technology as a tool to manipulate data and model equations.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can translate an application problem into a system of equations and solve it using the graphing utility.
3	The student can use a graphing calculator to solve a system of equations (or other problems) and interpret the solution.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: Matrices can help us more effectively model and solve linear systems.	EQ1: How do I know when to use matrices to solve linear systems?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 CEU2, CEQ2a, b, c EU1, EQ1 A.REI.8. C.9 N.VM.6, 9, 12 DOK 3	Students will organize data into a linear system of equations of three (or more) variables involving money invested in three different interest bearing accounts and the amount of interest earned. Students will model the system using a matrix and use technology to find and interpret the solution.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
additive identity additive inverse augmented matrix determinant coefficient matrix Gaussian elimination inverse matrix matrix multiplication minors multiplicative identity multiplicative inverse order of a matrix reduced row echelon form row and column row echelon form row operations scalar square matrix zero matrix	Construct a matrix equation to represent a system of linear equations (DOK 2)	*A.REI.8 Represent a system of linear equations as a single matrix equation.
	Solve a system of linear equations using inverse matrices (DOK 2) Solve a system of linear equations with three or more variables using technology (DOK 2)	*A.REI.C.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
	Use matrices to represent and manipulate data (DOK 2)	*N.VM.6 Use matrices to represent and manipulate data.
	Observe that matrix multiplication for square matrices is associative and distributive, but not commutative (DOK 1)	*N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
	Interpret the absolute value of the determinant in terms of area (DOK 2)	*N.VM.12 Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

PRE-CALCULUS
UNIT 4: Vectors and Parametric Equations (Optional) **SUGGESTED DURATION: 2 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will use vectors and parametric equations to model movement in the coordinate plane and the physical world.	
UNIT LEARNING SCALE	
4	In addition to level 3 performances, the student will create their own scenario that can be represented in vector form.
3	The student can: • write the component form of vectors; • perform operations on vectors; and • model problems involving quantities that can be represented by vectors and parametric equations.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: Vectors can be utilized to display the magnitude and direction of an object in a physical application.	EQ1: When will I use vectors to model physical quantities?
EU2: Curves defined parametrically not only tell us the position of a particle at a given time, but also display the direction of motion.	EQ2: When is it useful to model a situation parametrically and why?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 CEU2, CEQ2a, b, c EU1, EQ1 EU2, EQ2 N.VM.A.1, 2, 3 N.B.4, 4b N.VM.5, 10 DOK 3	Students will complete each independent assessment: 1. Students will use a resultant vector to determine the position of an object. 2. Students will determine required tension to maintain equilibrium. 3. Students will graph a set of parametric equations and indicate direction.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
component form direction direction angle dot product initial point magnitude orthogonal parameter parametric equations plane curve speed standard unit vectors terminal point unit vector vector vector projection velocity	Represent vectors as directed line segments (DOK 1) Use appropriate symbols for vectors and their magnitudes (DOK 1)	*N.VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lvert\lvert\boldsymbol{v}\rvert\rvert$, v).
	Find the components of a vector by subtracting the coordinates of the initial point from the coordinates of the terminal point (DOK 1)	*N.VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
	Model problems involving quantities that can be represented by vectors (DOK 3)	*N.VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.
	Add vectors using a variety of techniques such as graphing them end-to-end, using their components, and/or using the parallelogram rule (DOK 2) Represent vector subtraction graphically by connecting the tips in the appropriate order and using their components (DOK 2)	*N.VM.B.4 Add and subtract vectors.
	Determine the magnitude and direction of the sum of two vectors given the magnitude and direction of each (DOK 2) Explain why the magnitude of a sum of two vectors is typically not the sum of the magnitudes (DOK 2)	*N.VM.B.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
	Compute the magnitude of a scalar multiple cv using $\lvert\lvert cv\rvert\rvert = \lvert c\rvert v$ (DOK 2) Describe the direction of cv knowing that when $\lvert c\rvert v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$) (DOK 2)	*N.VM.5 Multiply a vector by scalar.
	Multiply a vector by a matrix to produce another vector (DOK 1)	*N.VM.10 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

PRE-CALCULUS
UNIT 5 : Polynomial and Rational Functions **SUGGESTED DURATION: 3-4 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will graph and analyze polynomial and rational functions in order to predict end behavior.	
4	In addition to level 3 performances, the student can interpret the meaning of the characteristics of polynomial and rational functions, as well as analyze the meaning of the zeros of polynomial functions.
3	The student can: • find the zeros of a polynomial using the fundamental theorem of algebra, factoring, and technology; • determine the end behavior of polynomial and rational functions; and • analyze the characteristics of polynomial and rational functions.
2	The student can: • find the zeros of a polynomial function but are not proficient in utilizing all the methods; and • analyze the characteristics of polynomial and rational functions with technology only.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: Polynomials and rational functions and their properties can be used to solve problems and make predictions.	EQ1a: How do restrictions (asymptotes) in rational functions represent real-world situations? EQ1b: Why do we need to predict end behavior?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 CEU2, CEQ2a, b, c EU1, EQ1a, b A.APR.B.3 A.APR.D.6, 7 F.IF.B.4 F.IF.C.7c, d DOK 3	Students will sketch and analyze graphs of functions by hand and utilize technology to approximate all extrema. Students will then describe the domain and characteristics of the function. Students will also discuss any discrepancies in relations to possible restrictions.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
asymptotes complex numbers conjugates continuity decreasing intervals Descartes' rule of signs domain end behavior factor theorem factoring fundamental theorem of algebra horizontal asymptotes imaginary numbers increasing Intervals intercepts leading coefficient test long division partial fraction decomposition quadratic equations radicals range rational functions rational root test relative extrema remainder theorem slant asymptotes synthetic division vertical asymptotes zeros	Divide polynomials (DOK 2) Given a polynomial p(x) and a number a, divide p(x) by (x-a) to find p(a), then apply the remainder theorem and conclude that p(x) is divisible by (x-a) if and only if p(a) =0 (DOK 3)	A.APR.B.2 Know and apply the remainder theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
	Create a sign chart for a polynomial f(x) using the polynomial's x-intercepts and testing the domain intervals for which f(x) greater than and less than zero (DOK 2) Use the x-intercepts of a polynomial function and the sign chart to construct a rough graph of the function (DOK 3)	A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
	Use multiple methods, including inspection, long division and in a computer algebra system to rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ (DOK 3)	A.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
	Add, subtract, multiply, and divide rational expressions (DOK 1) Informally verify that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression (DOK 3)	A.APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
	Graph polynomial functions, by hand or by using technology, show/label maxima and minima of the graph, identify zeros when suitable factorizations are available, and show end behavior (DOK 2)	F.IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
	Graph polynomial functions, by hand or by using technology, show/label asymptotes of the graph, identify zeros when suitable factorizations are available, and show end behavior (DOK 2)	F.IF.C.7d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
	Interpret key features of graphs and tables of functions in terms of the contextual quantities each function represents (DOK 2) Sketch graphs showing the key features of a function and modeling a relationship between two quantities, given a verbal description of the relationship (DOK 2)	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

PRE-CALCULUS
UNIT 6: Exponentials And Logarithms **SUGGESTED DURATION: 3 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will model, interpret, and make predictions about exponential and logarithmic relationships.	
UNIT LEARNING SCALE	
4	In addition to level 3 performances, the student can choose the appropriate function to model the given scenario.
3	The student can: • analyze the characteristics of exponential and logarithmic functions including asymptotes and end behavior; • model, interpret, and make predictions about exponential and logarithmic relationships; and • use properties of exponential and logarithmic functions to solve equations.
2	The student can: • state the characteristics of exponential and logarithmic functions • make predictions about exponential and logarithmic functions provided the equation is given; and • graph with the assistance of a graphing calculator.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: Growth and decay relationships can be modeled through exponential and logarithmic functions.	EQ1: When would I use exponential or logarithmic functions to make predictions?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 CEU2, CEQ2a, b, c EU1, EQ1 F.IF.C.7e LE.F.4 DOK 3	Students will model and make predictions about population growth using the appropriate exponential or logarithmic function.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
asymptotes base change of base formula domain end behavior exponential models exponential function exponential decay exponential growth graph of a function input-output table inverse function logarithm logarithmic function logarithmic models nonlinear models properties of logarithms range	Describe the inverse relationship between exponential and logarithmic functions, including equations and graphs (DOK 2) Solve exponential equations (DOK 2) Solve logarithmic equations that transfer into compound interest, half-life, and other real-world applications (DOK 3)	F.BF.B.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
	Graph exponential and logarithmic functions, by hand or by using technology, and show intercepts, end behavior, and asymptotes (DOK 2)	F.IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~
	Express the solution of an exponential model as a logarithmic equation (DOK 3)	F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where ab, b, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
	Interpret key features of graphs and tables of functions in terms of the contextual quantities each function represents (DOK 2) Sketch graphs showing the key features of a function and modeling a relationship between two quantities, given a verbal description of the relationship (DOK 2)	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

PRE-CALCULUS
UNIT 7: Series And Sequences **SUGGESTED DURATION: 2 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will analyze and create sequences and series that model the long-term behavior of situations involving sequential, arithmetic, or geometric change.	
UNIT LEARNING SCALE	
4	In addition to level 3 performances, the student can extend their knowledge of convergence and divergence to other series and tests.
3	The student can: • use the properties of arithmetic and geometric sequences and series to determine the general term, partial sums, and convergence/divergence; • use the binomial theorem and Pascal's triangle to expand a binomial with a positive integer power; • write and use recursive sequences; and • interpret and use summation notation.
2	The student can: • recognize arithmetic and geometric sequences; • use summation notation; and • write the general term with assistance.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU1: Functions and their properties can be used to model and analyze real-world situations to solve problems and make predictions.	CEQ1a: Can all relationships in the real-world be modeled with functions? CEQ1b: How can I use functions to predict real-world events? CEQ1c: How do I know which function will best model the scenario?
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: Series and sequences can be utilized to represent various patterns.	EQ1: How do I know if a scenario is best represented by a series or sequence?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 CEU1, CEQ1a, b, c CEU2, CEQ2a, b, c EU1, EQ1 A.APR.C.5, F.BF.A.1a, 2 F.IF.A.3 F.LE.A.2 DOK 3	Students will determine when an infinite series has a sum and find the sum, if possible.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
arithmetic sequence binomial coefficient binomial theorem factorial geometric sequence index of summation partial sums Pascal's triangle sequence series summation notation upper and lower limits	Define the binomial theorem and compute combinations (DOK 1) Apply the binomial theorem to expand $(x + y)^n$ when n is a positive integer and x and y are any number (DOK 2) Explain the connection between Pascal's triangle and the determination of the coefficients in the expansion of $(x + y)^n$, when n is a positive integer and x and y are any number (DOK 2)	A.APR.C.5 Know and apply the binomial theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
	Define explicit functions and recursive processes (DOK 1) Write a function that describes a relationship between two quantities by determining an explicit expression, a recursive process, or steps for calculation from a context (DOK 2)	F.BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
	Identify arithmetic and geometric patterns in given sequences (DOK 1) Determine the recursive rule given arithmetic and geometric sequences (DOK 1) Determine the explicit formula given arithmetic and geometric sequences (DOK 1) Justify the translation between the recursive form and explicit formula for arithmetic and geometric sequences (DOK 2) Generate arithmetic and geometric sequences from recursive and explicit formulas (DOK 2) Given an arithmetic or geometric sequence in recursive form, translate into the explicit formula and vice versa(DOK 2) Use given and constructed arithmetic and geometric sequences, expressed both recursively and with explicit formulas, to model real-world situations (DOK 2)	F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
	Construct linear functions, including arithmetic and exponential sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table) (DOK 2)	F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

PRE-CALCULUS
UNIT 8: Limits, Continuity, and Basic Derivatives (Optional - Applications of Derivatives) **SUGGESTED DURATION: 6-7 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will use limits to describe the instantaneous rate of change of functions and predict behavior.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can apply derivatives to authentic scenarios.
3	The student can: • evaluate and interpret limits; • use limits to determine continuity and classify discontinuity; • use derivatives to determine relative minimums and maximums of various functions; and • interpret the derivative as a rate of change.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: A limit can be used to understand the behavior of functions.	EQ1: Why do we study limits? What do they tell us?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 CEU2, CEQ2a, b, c EU1, EQ1 A.APR.3 F.BF.1 F.IF.1, 2, 4, 6, 8 DOK 3	Students will complete each independent assessment: 1. Students will interpret the derivative of a function represented graphically, numerically or analytically to determine the behavior of the function and justify their conclusions referencing appropriate tests. 2. Students will analyze the relationship between an object's position, velocity, and acceleration function to find various information such as the position of the object when its velocity is constant, when the object changes direction, and when the object's speed is increasing. 3. Students will use derivatives to find the minimum amount of material to be used to create a rectangular shaped box with a specific volume and justify their conclusion.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
continuity derivative discontinuity • point • jump • infinite • oscillating equation of a tangent line first derivative test infinite limit instantaneous rate of change limit limit does not exist one-sided limit point of inflection second derivative test slope of a secant line **Optional:** velocity acceleration optimization	Write a function that models a real-world situation by determining an explicit expression and using a derivative to find the maximum or minimum of the function (optimization) (DOK 3)	F.BF.1 Write a function that describes a relationship between two quantities.
	Identify the domain of various functions including piecewise, rational, and trigonometric for the purpose of discussing where the function is continuous (DOK 2) Identify where a function is discontinuous given its graph or its equation, and classify it as infinite, removable, jump or oscillating (DOK 2) Define continuity as when a function's value is equal to the limit value (DOK 2)	F.IF.1Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
	Interpret a limit as a function's intended output for a given input. Use direct substitution to evaluate limits when appropriate (DOK 2)	F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
	Use a graph or knowledge of a function's end behavior to evaluate infinite limits (DOK 2) Interpret key features including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity (DOK 2)	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
	Calculate the average rate of change of a function (presented symbolically or as a table) over a specified interval (DOK 2) Apply the limit definition of a derivative to find the instantaneous rate of change at a point (DOK 2)	F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
	Write functions in equivalent forms by factoring, multiplying by a conjugate, or simplifying a complex fraction and evaluating limits analytically when direct substitution fails (DOK 2)	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
absolute extrema relative extrema sign line	Create a sign chart for a polynomial's first and second derivatives using x-values for which the derivative is either zero or undefined and testing the domain intervals for which f'(x) and f''(x) is greater than and less than zero (DOK 2) Use the sign chart to construct a rough graph of the function, identifying relative extrema and points of inflection (DOK 3)	A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

PRE-CALCULUS
UNIT 9: Conics (Optional) **SUGGESTED DURATION: 0-2 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will connect conic sections to their quadratic forms in order to create graphical models.	
UNIT LEARNING SCALE	
4	In addition to level 3 performances, the student can determine which conic section is appropriate for a given scenario.
3	The student can: • graphically analyze the properties of each conic section; • identify the similarities and differences between the conic sections.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in level 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
CEU2: The transfer of knowledge and perseverance are necessary when utilizing problem solving techniques to reach a solution.	CEQ2a: What techniques can I use to persevere through solving a problem? CEQ2b: When are multiple solutions or problem solving techniques appropriate? CEQ2c: How do I determine the most efficient method to solve a problem?
EU1: Conics and their properties create a graphical representation to provide additional insight into the behavior of non-functions.	EQ1: Why do we study conic sections?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 CEU2, CEQ2a, b, c EU1, EQ1 G.GPE.A.1,2, 3 DOK 3	Students will use a given quadratic form to determine the conics section and create a graphical model.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
asymptotes axis circle conic section conjugate axis degenerate conic directrix ellipse focus hyperbola major axis minor axis parabola transverse axis vertex	Derive the equation of a circle using the Pythagorean theorem, given the coordinates of the center and length of the radius (DOK 2) Determine the center and radius of a circle by completing the square (DOK 2)	G.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean theorem; complete the square to find the center and radius of a circle given by an equation.
	Describe a parabola including the relationship of the focus and the equation of the directrix to the parabolic shape (DOK 1) Derive the equation of a parabola given the focus and the directrix (DOK 2)	G.GPE.A.2 Derive the equation of a parabola given a focus and directrix.
	Derive the equations of the ellipses given the foci, using the fact that the sum of distances from the foci is constant (DOK 2) Derive the equations of the ellipses given the foci, using the fact that the sum of distances from the foci is constant (DOK 2) Use definitions of conic sections and the Pythagorean theorem to derive equations (DOK 3)	G.GPE.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.