

**FREEHOLD REGIONAL HIGH SCHOOL DISTRICT**

**OFFICE OF CURRICULUM AND INSTRUCTION**

**MATHEMATICS DEPARTMENT**

# **MODERN MATH**

Grade Level: 11-12

Credits: 5

**BOARD OF EDUCATION ADOPTION DATE:**

**AUGUST 31, 2015**

[SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING](#)

APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS

APPENDIX B: ASSESSMENT EVIDENCE

APPENDIX C: INTERDISCIPLINARY CONNECTIONS

# **FREEHOLD REGIONAL HIGH SCHOOL DISTRICT**

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## MODERN MATH

### COURSE PHILOSOPHY

Modern world challenges require a shift in instruction away from routine manipulation of symbols and procedures toward an in-depth conceptual understanding of mathematics. Explorations of a variety of topics will broaden student perspective of mathematical applications beyond traditional mathematics courses and offer students the opportunity to apply mathematics in real-world scenarios and functions.

### COURSE DESCRIPTION

*Modern Math* is intended to introduce mathematical fields of study that go beyond Algebra and Geometry. This course opens the door for students to pursue passions and interests that require a deeper understanding of mathematics. *Modern Math* stresses the importance of the recurring themes and patterns in mathematics, which become evident in the context of modern world scenarios. Building on their Algebra I and Geometry foundations, students will think numerically, algebraically, spatially, and logically by engineering solutions to current dilemmas through the Standards of Mathematical Practice. After completing this course, students will be equipped to pursue several rigorous courses in mathematics on the high school and collegiate levels.

## COURSE SUMMARY

### COURSE GOALS

CG1: Students will make informed decisions in today's information-based world by analyzing data and justifying their conclusions.

CG2: Students will apply the mathematics they know to solve problems arising in the modern world.

CG3: Students will make informed predictions and synthesize their understanding of mathematics to independently generate and test their predictions mathematically.

CG4: Students will model mathematics symbolically and graphically.

### COURSE ENDURING UNDERSTANDINGS

CEU1: The way data is collected, organized and displayed influences interpretation.

CEU2: Problem-solving requires the ability to define the problem, identify critical information, choose an appropriate method, and support reasoning with evidence.

CEU3: The results of a mathematical investigation can be used to predict events and support or refute an argument.

CEU4: Real-world situations can be represented symbolically and graphically.

### COURSE ESSENTIAL QUESTIONS

CEQ1: How can the same data lead to different conclusions?

CEQ2: What makes your problem-solving strategy effective and efficient?

CEQ3: How can mathematics help us make decisions?

CEQ4: How can mathematics be visualized? Is there one best way to do this?

**UNIT GOALS & PACING**

UNIT TITLE	UNIT GOALS	RECOMMENDED DURATION
<a href="#">1: Introduction to Discrete Math</a>	Students will analyze real-world scenarios and improve efficiency through planning, routing, social choices, and scheduling techniques.	9 weeks
<a href="#">2: Introduction to Computer Science</a>	Students will create a computer program to present data, to solve problems and/or model scenarios.	8 weeks
<a href="#">3: Introduction to Probability and Statistics</a>	Students will independently evaluate the role of probability and statistics in modeling everyday scenarios and in the decision-making process.	8 weeks
<a href="#">4: Functions and Applications</a>	Students will utilize linear, quadratic, and other functions to model real-world situations, solve problems, communicate their answers, and justify their reasoning.	9 weeks

**MODERN MATH****UNIT 1: Introduction to Discrete Math****SUGGESTED DURATION: 9 weeks****UNIT OVERVIEW****UNIT LEARNING GOALS**

Students will analyze real-world scenarios and improve efficiency through planning, routing, social choices, and scheduling techniques.

**UNIT LEARNING SCALE**

4	In addition to level 3 performances, the student can analyze the work of a peer and critique and/or make improvements or suggestions.
3	The student can: <ul style="list-style-type: none"> <li>• identify, create, and analyze paths and circuits using different methods (Euler, Hamilton);</li> <li>• determine and justify when a solution is optimal;</li> <li>• create and analyze visual representations of data using different methods (map coloring, scheduling);</li> <li>• identify and analyze winners using different methods of social choice (fair division, voting methods);</li> <li>• communicate and justify reasoning.</li> </ul>
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance or makes larger errors in attempting to reach level 3.
0	Even with help, the student does not exhibit understanding of performances listed in level 3.

**ENDURING UNDERSTANDINGS****ESSENTIAL QUESTIONS**

EU1: It is possible to find the best solution within given constraints using methods of optimization.	EQ1: How do I determine if a solution is optimal?
EU2: Algorithms can effectively and efficiently be used to quantify and interpret discrete information.	EQ2: How can algorithmic thinking be used to solve problems?
EU3: Various representations can be used to communicate thinking and solve problems.	EQ3: How do you choose what representation is best to communicate your ideas?
EU4: Various methods can yield different results when solving a problem or finding a winner.	EQ4: How do I determine the best method when each method yields different result?

**NJCCCS & COMMON CORE STANDARDS**




N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.






N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU 1, 2, 3 EQ 1, 2, 3 N.Q.1, 2, 3 SMP 1, 2, 4 DOK 3	The students will take the role of mayor of the small town of Discreteville. Students will be given a map of the town with specific locations and parameters (e.g., number of homes, required buildings such as a post office or a school). Students will need to create a route to plow snow from the streets. Students will use Euler circuits to find the optimal and most cost efficient route. Students must also find the most cost efficient method to deliver packages by utilizing the Hamilton method. Students will be required to create a detailed modern town flag while minimizing the number of colors used (map coloring). Students will also have to create a work schedule for the town office, given a specified number of employees and job descriptions. Finally, students will interpret data generated from an official election and determine the winner using several voting methods.

SUGGESTED STRATEGIES		
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
<p>Students will be given several vertex edge graphs and be required to identify each graph as an Euler circuit, Euler path, or neither. After classifying each graph, students will be required to find the Euler circuits and Euler paths.</p> <p> Provide students with graphs of various levels of difficulty based on the number of vertices and edges.</p>	<p>Circuits:</p> <ul style="list-style-type: none"> <li>• vertex</li> <li>• edge</li> <li>• graph</li> <li>• circuit</li> <li>• path</li> <li>• Euler</li> <li>• Hamilton</li> <li>• weighted edge</li> <li>• minimal</li> <li>• nearest-neighbor</li> <li>• brute-force</li> </ul>	<p>Create and analyze paths and circuits using different methods</p> <p>Determine and justify when a solution is optimal</p> <p>Create and analyze visual representations of data</p> <p>DOK 2, 3</p>
<p>Students will be required to color several pictures using the least number of colors possible, but must adhere to the rules of map coloring and the four color theorem.</p> <p> Use the complexity of the picture to vary the level of difficulty.</p> <p> Students can utilize dynamic geometric software or paint software to color their pictures. Students can utilize web based coloring sites to assist them with coloring and choosing pictures.</p>	<p>Map Coloring:</p> <ul style="list-style-type: none"> <li>• adjacent</li> <li>• edge</li> <li>• vertices</li> <li>• minimum</li> <li>• four color theorem</li> </ul>	<p>Minimize the number of colors necessary to color a map/diagram</p> <p>Minimize the number of colors necessary to color vertices in a given graph</p> <p>Determine and justify when a solution is optimal</p> <p>DOK 3, 4</p>

## SUGGESTED STRATEGIES

ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
<p>Students will be managers at a retail store. They will be required to create a weekly schedule based upon their employee's availability and positions that need to be filled.</p> <p> Vary the type of store and thus the number of workers. For example, a bagel store only requires a cashier and a food preparer, retail store requires a cashier, floor person, and stock person and a restaurant requires a hostess, multiple servers, and multiple cooks.</p> <p> Students can utilize computer based spreadsheets to create and organize their schedules.</p>	<p>Scheduling:</p> <ul style="list-style-type: none"> <li>• logic</li> <li>• minimal</li> <li>• optimal solutions</li> </ul>	<p>Create and analyze a schedule involving multiple people and processes</p> <p>Minimize the number of employees required to fill all necessary positions</p> <p>Determine and justify when a solution is optimal</p> <p>DOK 3, 4</p>
<p>Students will be given many different objects that have different values to different people (e.g., different size and types of candy bars, different types/colors of pens). In small groups, students will use the Method of Markers to determine the items that each student will earn. Students will determine the best method to distribute the leftover items amongst their group.</p> <p> Increase the number of items and/or the number of students in a group.</p>	<p>Fair Division:</p> <ul style="list-style-type: none"> <li>• equal objects</li> <li>• unequal objects</li> <li>• participants</li> <li>• Method of Markers</li> <li>• apportionment</li> </ul>	<p>Create and solve fair division problems using the Method of Markers</p> <p>Create and solve fair division examples using apportionment</p> <p>Determine and justify when a solution is optimal</p> <p>DOK 3, 4</p>
<p>Students will survey a specified number of people to create a preference list on a pop culture/current event. Students will be required to decide the winner(s) using all of the voting methods learned in class.</p> <p> Increase the number of items in the preference list and/or increase the number of people surveyed. Students can create their own survey question or teachers can supply pre-created survey questions/answers/data.</p> <p> Students can utilize online/graphing calculator randomizer to create the preference list.</p>	<p>Voting/Social Choices:</p> <ul style="list-style-type: none"> <li>• candidates</li> <li>• preference list</li> <li>• ranking</li> <li>• majority</li> <li>• plurality</li> <li>• run-off</li> <li>• Borda Count Method</li> <li>• Condorcet Method</li> </ul>	<p>Decide the winner of voting examples using the different methods of voting</p> <p>Determine and justify when a solution is optimal</p> <p>Determine and justify why a method is efficient</p> <p>DOK 3, 4</p>

**MODERN MATH****UNIT 2: Introduction to Computer Science****SUGGESTED DURATION: 8 weeks****UNIT OVERVIEW****UNIT LEARNING GOALS**

Students will create a computer program to present data, to solve problems and/or model scenarios.

**UNIT LEARNING SCALE**

4	In addition to level 3 performances, students can assist peers in debugging their programs.
3	The students can: <ul style="list-style-type: none"> <li>demonstrate understanding of logic in programming;</li> <li>design algorithms to solve problems;</li> <li>translate algorithms into coded form;</li> <li>use computer applications to solve problems.</li> <li>create and design their own original computer program.</li> </ul>
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance or makes larger errors in attempting to reach level 3.
0	Even with help, the student does not exhibit understanding of performances listed in level 3.

**ENDURING UNDERSTANDINGS****ESSENTIAL QUESTIONS**

EU1: There are multiple avenues to design solutions to problems and some may be more efficient than others.

EQ1a: How can you determine the best algorithm to solve a problem?

EQ1b: Can a program be effective while not efficient?

EU2: Writing a computer program requires the consideration of the specifications as well as predicting for unforeseen events.

EQ2: What determines the use of decision making in program design?

EU3: A computer executes instructions exactly as written, not necessarily as intended.

EQ3: How can a program function correctly, but not produce the intended output?

**NJCCCS & COMMON CORE STANDARDS****NJCCCS:**

8.1.12.B.1 Design and pilot a digital learning game-(program) to demonstrate knowledge and skills related to one or more content areas or a real-world situation.

**CCCS:**

G.MG.3 Apply geometric methods to solve design problems.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

N.Q.3 Choose a level of accuracy to limitations on measurement when reporting quantities.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.


F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ .

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-BF.1.a Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.



COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU 1, 2, 3 EQ 1, 2, 3 8.1.12.B.1 G.MG.3 N.Q.2., 3 A.SSE.2 F.IF.1, 2 F-BF.1 SMP 1, 3, 4, 5 DOK 4	Students will develop an independent program utilizing Scratch to demonstrate their knowledge of the tools they've learned. The project should include a set number of the following design specifications: sprites, costumes, stage, motion, loops, broadcasting and receiving messages, variables, query, and story/experience. Students will have to use their troubleshooting skills to make small adjustments to fix any errors. Students will answer questions about their program including the overall purpose, the story it tells, and the experience it provides. Students will "pitch" their program in 1-3 sentences, convincing users to try it!

SUGGESTED STRATEGIES		
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
Students will create programs outlined in the Scratch curriculum guide.   Modify this by requiring all components or a select few.	Programming Terminology: <ul style="list-style-type: none"> <li>• coding</li> <li>• algorithms</li> <li>• data structure</li> <li>• applications</li> <li>• sequence</li> <li>• loops</li> <li>• parallelism</li> <li>• events</li> <li>• conditionals</li> <li>• operators</li> <li>• computational platform</li> </ul>	Create algorithms  Design and implement programs  Debug programs  DOK 2, 3, 4

**MODERN MATH****UNIT 3: Introduction to Probability and Statistics****SUGGESTED DURATION: 8 weeks****UNIT OVERVIEW****UNIT LEARNING GOALS**

Students will independently evaluate the role of probability and statistics in modeling everyday scenarios and in the decision-making process.

**UNIT LEARNING SCALE**

4	In addition to level 3 performances, the student can critique and justify the work of others.
3	The student can: <ul style="list-style-type: none"> <li>• use statistics as a process for making inferences;</li> <li>• justify conclusions and support viable argument (e.g., create and perform an experiment to justify or refute theoretical evidence);</li> <li>• use statistical data to make everyday decisions;</li> <li>• find the probability of an event (e.g., coins, cards, dice);</li> <li>• use probability to justify and make fair decisions.</li> </ul>
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance or makes larger errors in attempting to reach level 3.
0	Even with help, the student does not exhibit understanding of performances listed in level 3.

**ENDURING UNDERSTANDINGS**

EU1: Statistics is necessary to make accurate decisions involving data.

EU2: Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.

EU3: Graphical displays should be based on the type of data, the audience and the intended message.

**ESSENTIAL QUESTIONS**

EQ1: How does statistics play a role in our everyday life?

EQ2: When is it appropriate to use probability to make decisions?

EQ3a: How do you determine the best graphical display for your message?

EQ3b: How do people use data and graphical displays to influence others?

**NJCCCS & COMMON CORE STANDARDS**

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; ~~develop a margin of error through the use of simulation models for random sampling~~

S.IC.6 Evaluate reports based on data.



S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or”, “and”, “not”).





S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.



S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more ~~different data sets.~~

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU 1, 2, 3 EQ 1, 2, 3a, 3b S.IC.1 S.IC.2 S.IC.4 S.IC.6 S.CP.2 S.ID.2 S.MD.6: SMP 3, 4, 8 DOK 3	<p>Provide students with the following scenario: <i>Your advertising and marketing agency has just heard from three new potential clients. You and your team must choose which client you would like to take on and create a great marketing campaign for them. Analyze the variables involved, conduct any necessary experiments or surveys, and come up with a selling point. If other teams in the ad agency pick the same client, you will need to compete against them for the client's account.</i></p> <ol style="list-style-type: none"> <li><i>Video games are getting a bad rap in some places as time-wasters and addictions, but there must be some good that kids get out of them. The client is a new retail store selling video games. What is a good selling point that is backed by data?</i></li> <li><i>Some young people don't like to play sports but they need to "get physical" to stay healthy. The client is a new gym in town whose target market consists of high school and college students. What positive effects of staying fit, backed by data, can you sell to these young people?</i></li> <li><i>A health food store whose motto is "You are what you eat" wants your help in getting young people to understand the importance of eating right. How well or poorly do young people eat on a weekly basis? For what reasons? What might be the best selling point for the store, based on survey and scientific data?</i></li> </ol>

SUGGESTED STRATEGIES		
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
<p>Provide the students with the following scenario:</p> <p><i>You are on the game show "Let's Make A Deal" looking at three doors. Behind one is a brand-new car. Behind the others are gag gifts. You get to choose one door. If it's the car, you win! You choose Door #1. But before the host unveils its contents, he opens Door #2 and reveals a gag gift. You may have picked the right door! Then, the host asks if you'd like to make a deal. If you want, you can switch to Door #3. What would switching do: improve your odds, worsen your odds, or leave them the same?</i></p> <p>Let the student's debate in small groups and justify their answers with evidence.</p> <p> Incorporate more doors with different prizes (some gag gifts and some not) to see how the probabilities would be affected.</p> <p> Students can use an online simulation for "Let's Make a Deal" to perform experimental probabilities and test their theoretical probabilities.</p>	<p>Probability:</p> <ul style="list-style-type: none"> <li>theoretical probability</li> <li>favorable outcomes (successes)</li> <li>sample space (possible outcomes)</li> <li>experimental probability (simulation)</li> <li>odds</li> <li>chance</li> <li>event</li> <li>probability distribution</li> <li>probability histogram</li> <li>tree diagrams</li> </ul>	<p>Calculate theoretical probability</p> <p>Analyze calculations and make inferences</p> <p>Use technology to perform a simulation to test theoretical probabilities</p> <p>Compare experimental and theoretical probability with respect to sample size</p> <p>DOK 2, 3</p>

SUGGESTED STRATEGIES		
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
<p>Game of Chance: Students will work in pairs. Ask the students, “How many different outcomes are there when you roll two dice?” Students will then be asked “How many ways are there to roll a sum of two using two dice?” Continue asking these questions for the sums of three to twelve and have students chart their responses. Students will work with their partners to calculate these probabilities. Students will be asked probing questions about their findings such as “If you are playing a game with two dice, why would ‘7’, ‘2’, and ‘12’ be the most valuable rolls in the game?”</p> <p> Once students understand this concept they should explore 'expected value' for games of chance or create their own game.</p> <p> Introduce a third die to calculate theoretical and experimental probabilities.</p> <p> Students will perform the experimental probability of rolling two dice using the dice simulator. Students will “roll” several times, record their data, and make conclusions about their experimental data.</p>	<p>Probability:</p> <ul style="list-style-type: none"> <li>• theoretical probability</li> <li>• experimental probability</li> <li>• expected value</li> <li>• favorable outcomes</li> <li>• possible outcomes</li> <li>• trials</li> <li>• ratio</li> <li>• data</li> <li>• sample space</li> <li>• event</li> <li>• independent events</li> <li>• mutually exclusive(disjoint) events</li> <li>• complement of an event</li> </ul>	<p>Compare and contrast experimental and theoretical probabilities</p> <p>Calculate expected values</p> <p>Explain whether an event is independent, disjoint, or neither</p> <p>DOK 2, 3</p>
<p> Interpreting World Statistics: Students use statistics to examine the effects of global events. For example, students can examine the impact of World War II. Use a mapping tool such as Gapminder.</p>	<p>Statistics:</p> <ul style="list-style-type: none"> <li>• statistical data representations</li> <li>• graphs</li> <li>• frequency</li> <li>• distribution</li> <li>• patterns in distribution</li> <li>• time plots</li> <li>• linear regression</li> <li>• modeling</li> <li>• influential points</li> <li>• outliers</li> <li>• extrapolation</li> <li>• correlations</li> </ul>	<p>Analyze data</p> <p>Look for trends in data</p> <p>Look for deviations and explain</p> <p>Explain relationships between time and variables in context</p> <p>Use data to make predictions and discuss confidence in predications</p> <p>DOK 2, 3</p>

SUGGESTED STRATEGIES		
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
<p>Students will use technology to research two major cities and their history of temperatures for thirty years. They will calculate the mean, median and mode for their data. They will also calculate the average temperature for five year intervals during the thirty years. Students will create a line graph to represent their data for the five year intervals and a histogram for their measures of central tendency, and calculate the variance and the standard deviation.</p> <p>Students will draw a normal curve from their data and find the percent of data that is within one, two, and three deviations from the mean. Students will determine if their curve is normal and answer the following questions:</p> <ul style="list-style-type: none"> <li>• What is the correlation between your curve to the normal curve?</li> <li>• Why do you think this is?</li> <li>• What conclusions can be drawn from your curve?</li> <li>• What estimations can be made about future data from your curve?</li> </ul> <p>Overall, students will draw conclusions based on their data and graphs and make inferences about their results.</p> <p> Students can come up with their own topic to research data, calculate measures of central tendency, and create visual representations.</p> <p> Have students use graphing calculators or graphing software to input data and create line graphs, histogram, and normal curve.</p>	<p>Statistics:</p> <ul style="list-style-type: none"> <li>• mean</li> <li>• median</li> <li>• mode</li> <li>• range</li> <li>• central tendency</li> <li>• outliers</li> <li>• data</li> <li>• histogram</li> <li>• line graph</li> <li>• bar graph</li> <li>• intervals</li> <li>• x- and y-axis</li> <li>• bell curve</li> <li>• data</li> <li>• frequency</li> <li>• conclusion</li> <li>• Normal distribution</li> <li>• Standard deviation</li> <li>• IQR</li> <li>• correlation</li> </ul>	<p>Record data in tables</p> <p>Calculate measures of central tendency along with variance and standard deviations</p> <p>Compare and contrast central tendencies given specific data</p> <p>Organize data using different graphs, tables, and charts</p> <p>Find probabilities based on distributions</p> <p>Draw conclusions based on data and visual representations</p> <p>DOK 3</p>

**MODERN MATH****UNIT 4: Functions and Applications****SUGGESTED DURATION: 9 weeks****UNIT OVERVIEW****UNIT LEARNING GOALS**

Students will utilize linear, quadratic, and other functions to model real-world situations, solve problems, communicate their answers, and justify their reasoning.

**UNIT LEARNING SCALE**

4	In addition to level 3 performances, the student can model real-world situations and apply the concepts from level 3.
3	The student can: <ul style="list-style-type: none"> <li>graph quadratic equations in different forms (standard, vertex, intercept);</li> <li>identify vertex, axis of symmetry, intercepts, domain/range of quadratic functions;</li> <li>solve quadratic equations by graphing;</li> <li>solve systems involving linear and quadratic equations by graphing;</li> <li>graph exponential and rational functions;</li> <li>interpret and justify their reasoning</li> </ul>
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance or makes larger errors in attempting to reach level 3.
0	Even with help, the student does not exhibit understanding of performances listed in level 3.

**ENDURING UNDERSTANDINGS****ESSENTIAL QUESTIONS**

EU1: Real-world applications and solutions may be derived from linear representations.

EQ1: What real-world scenarios have a linear relationship?

EU2: Quadratic functions can be represented in a variety of ways. You can make connections from the components of a given equation and its represented graph.

EQ2a: How can we determine the best method to mathematically represent a quadratic function?

EQ2b: How is the graph of a quadratic function related to the algebraic model?

**NJCCCS & COMMON CORE STANDARDS**

A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.



## NJCCCS & COMMON CORE STANDARDS





- F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)t$ ,  $y = (0.97)t$ ,  $y = (1.01)12t$ ,  $y = (1.2)t/10$ , and classify them as representing exponential growth or decay.
- F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- F.BF.1 Write a function that describes a relationship between two quantities.
- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
  - c. (+) Compose functions. For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.

## COMMON ASSESSMENT

ALIGNMENT	DESCRIPTION
LG1 EU 1, 2 EQ 1, 2a, EQ 2b F-IF.7,F-IF.8 SMP 4, 8 DOK 2	Create a picture or design using transformed functions. There should be one linear, quadratic, rational, and exponential function, and transformations of each of those functions. All transformations must be represented in the picture. On a separate sheet, you will include a mathematical description of each transformation and a description of your picture. The project will be graded on the composition complexity of the transformations, the visual appeal/creativity, and the mathematical accuracy.

## SUGGESTED STRATEGIES

ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
Students will be given different functions to graph. They must determine the type of function and the best method for graphing. Students will justify their reasoning.	linear quadratic rational exponential functions vertex intercepts axis-of-symmetry domain range asymptotes standard form vertex form intercept form	Determine which type of function the equation represents and how to graph it  Graph linear, quadratic, rational, and exponential functions  Identify parts of the function from the graph  DOK 2
 Students can create their own functions (linear, quadratic, rational, and exponential).		
 Students will use graphing calculators or graphing software to compare graphs of different functions and check their work.		

SUGGESTED STRATEGIES		
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE
<p>Students will transform several parent graphs using given specifications and display these transformations using equations, graphs, and function notation.</p> <p> Students can create their own transformations for each function.</p> <p> Students use graphing calculators or graphing software to graph the functions.</p>	<p>parent function linear quadratic rational exponential translation reflection vertical/horizontal stretch/shrink</p>	<p>Perform transformations on different types of functions</p> <p>Represent transformations through equations, function notation, and graphs</p> <p>DOK 2</p>
<p>Students will be given a variety of scenarios that can be modeled by quadratic functions. Students must identify the parts of the quadratic function, graph the function, and solve the problem.</p> <p> Students can create and illustrate different situations that can be modeled by a quadratic function and graph.</p> <p> Students use graphing calculators or graphing software to graph the quadratic functions.</p>	<p>quadratic function parabola vertex intercepts</p>	<p>Graph quadratic functions</p> <p>Find the vertex and intercepts of quadratic functions</p> <p>Interpret different parts of the quadratic function in the context of the situation</p> <p>DOK 2</p>