FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

INTERNATIONAL BACCALAUREATE PROGRAM

MATHEMATICS STUDIES SL, YEAR 1

Grade Level: 11

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 29, 2016

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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IB MATHEMATICAL STUDIES SL, YEAR 1

COURSE PHILOSOPHY

The International Baccalaureate Organization provides the following philosophy for the teaching of mathematics and Mathematics Studies SL: *"The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: visual artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives, with all its interdisciplinary connections, provides a clear and sufficient rationale for making the study of this subject compulsory for students studying the full diploma.*

Mathematical Studies has an emphasis on applications of mathematics, and the largest section is on statistical techniques. It is designed for students with varied mathematical backgrounds and abilities. It offers students opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. . . Students taking this course are well prepared for a career in social sciences, humanities, languages or arts. These students may need to utilize the statistics and logical reasoning that they have learned as part of the mathematical studies SL course in their future studies."

COURSE DESCRIPTION

The International Baccalaureate Organization provides the following description for Mathematics Studies SL: "*The course syllabus focuses on important mathematical topics that are interconnected. The syllabus is organized and structured with the following tenets in mind: placing more emphasis on student understanding of fundamental concepts than on symbolic manipulation and complex manipulative skills; giving greater emphasis to developing students' mathematical reasoning rather than performing routine operations; solving mathematical problems embedded in a wide range of contexts; using the calculator effectively. The course includes project work, a feature unique to mathematical studies SL within group 5. Each student completes a project, based on their own research; this is guided and supervised by the teacher. The project provides an opportunity for students to carry out a mathematical study of their choice using their own experience, knowledge and skills acquired during the course. This process allows students to take sole responsibility for a part of their studies in mathematics."*

COURSE SUMMARY	
COURSE GOALS	
CG1: Students will synthesize historical perspectives and moral, social, and ethical implications using the graphical and numerical concepts of algebra, geometry, and trigonometry. CG2: Students will analyze how data and statistical literature either prove or disprove developments in society, and how this influences people's predictions and conclusions. CG3: Students will clearly communicate using mathematical models and differential calculus to demonstrate the principles of nature and math. CG4: Students will create solutions to real world problems by employing patience, persistence, initiative, and creative thinking.	
COURSE ENDURING UNDERSTANDINGS	COURSE ESSENTIAL QUESTIONS
CEU1: Relationships and functions can be represented numerically, graphically, and algebraically.	CEQ1: When is one representation of a function more useful than another?
CEU2: The way data is collected, organized, and displayed influences interpretation.	CEQ2a: How can the same data lead to different conclusions? CEQ2b: Can analyzed statistical data change the opinions of an entire data set or can opinions skew data asymmetrically?
CEU3: Mathematics can be utilized to model real world scenarios and make informed decisions.	CEQ3a: How can math be used to make informed decisions on a global level? CEQ3b: How have specific societies changed as a result of mathematical discoveries?
CEU4: Practicing mathematics improves our ability to make decisions in problem solving. It also allows us to find patterns to analyze situations in a systematic way.	CEQ4a: How can previous experiences lead us to making correct decisions in how to apply mathematical knowledge? CEQ4b: What can patterns tell us, and how does that recognition of pattern lead us to appropriate problem solving?

UNIT GOALS & PACING		
UNIT TITLE	**UNIT GOALS**	**RECOMMENDED DURATION**
Unit 1: Numbers & Algebra	LG1: Students will utilize linear equations, functions, measurement, and postulates to model real world applications, solve simultaneous problems, communicate their answers, and justify their reasoning. LG2: Students will analyze and represent sequences and series that model the long-term behavior of situations involving sequential change.	6 – 8 weeks
Unit 2: Geometry & Trigonometry	LG1: Students will analyze and solve geometric problems in a real world context by applying postulates and understanding the geometric properties. LG2: Students will solve application-based problems by extending the domain of trigonometric functions using the unit circle and modeling periodic phenomena.	8 - 10 weeks
Unit 3: Logic, Sets, & Probability	LG1: Students will use the laws of logic to analyze scenarios and make predictions. LG2: Students will use probability to analyze real world scenarios and make informed decisions.	10 - 12 weeks
Unit 4: Introduction to Mathematical Research	Students will investigate and research a problem in today's society and justify a solution mathematically.	2-4 weeks

IB MATHEMATICAL STUDIES SL, YEAR 1
UNIT 1: Numbers & Algebra **SUGGESTED DURATION: 6 – 8 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
LG1: Students will utilize linear equations, functions, measurement, and postulates to model real world applications, solve simultaneous problems, communicate their answers, and justify their reasoning. LG2: Students will analyze and represent sequences and series that model the long-term behavior of situations involving sequential change.	
UNIT LEARNING SCALE (LG1)	
4	In addition to score 3 performances, the student can create their own functions and postulates to model specific real world applications and explain their reasoning for the results obtained.
3	The student can: • employ appropriate methods to solve, graph, and interpret solutions of linear equations and functions; • use graphical representations of linear functions and technology to model real world applications; and • analyze solutions of simultaneous solutions to make determinations about future values.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of performances listed in score 3.
UNIT LEARNING SCALE (LG2)	
4	In addition to score 3 performances, the student can create their own analysis and representation of sequences that model the long-term behavior of situations involving sequential change and explain their reasoning for their independent thought.
3	The student can: • analyze and represent arithmetic and geometric sequences; • analyze and represent series; • model long-term behavior involving sequential change so one could determine the general outcome; and • create and solve exponential functions involving financial change.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Mathematical models can be used to describe and represent various physical relationships numerically, algebraically, and graphically.	EQ1a: Why are units important when solving problems? EQ1b: How does the tool of measurement limit the accuracy of the quantity measured? EQ1c: How are equations used to describe numbers or relationships and solve problems? EQ1d: What are the steps and strategies to justify a solution to a problem?
EU2: There are a variety of methods to solve and analyze polynomial equations and graphs and one method may be better than another given the polynomial.	EQ2a: Why can one method be better than another to solve a problem? EQ2b: How do you determine the best method? EQ2c: Is the best method to solve a problem always the most efficient?

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU3: Sequential and series mathematics can be used to analyze patterns in both society and science.	EQ3a: How can a function help me understand the world around me? EQ3b: What arithmetic and geometric sequences can be used to model situations? EQ3c: Can everything in nature be explained mathematically? EQ3d: Does all data have a pattern? EQ3e: What do the symbols (parentheses, brackets, braces) represent when evaluating an expression?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1, 2 EU 1, EQ1d EU3, EQ3b N.RN.3 F.IF.3 F.BF.2 SMP 1, 2, 3, 4, 8 DOK 4	Students will be given a set of odd integers, and the theorem on how to obtain such numbers. Students will discuss if the statement is true for all future positive integers, and to justify their answer mathematically. Students will be expected to create their own theorem, justify their findings, and find an application in real life where it can be implemented.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
arithmetic sequences arithmetic series geometric sequences geometric series series sum of a finite arithmetic series sum of finite geometric series	Generate and display sequences in several ways, including explicit and recursive functions (DOK 3) Find the sums of finite arithmetic and finite and infinite geometric series (DOK 2)	A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
laws of algebra measurement number properties	Identify numbers (e.g, natural, integers, rational, irrational, real) (DOK 1) Apply measurement conversion, and accuracy of measurement a variety of ways (DOK 3) Explain error and percent error (DOK 3)	N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
diagrams domain functions mapping range	Explain function notation (DOK 2) Apply linear models utilizing function notation through a variety of means (DOK 3)	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range A.REI.3 Solve linear equations in one variable, including coefficients represented by letters.
systems of linear equations	Solve systems of equations (DOK 3)	A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a *system* with the same solutions. A-REI.C.6 Solve *systems* of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

IB MATHEMATICAL STUDIES SL, YEAR 1
UNIT 2: Geometry & Trigonometry **SUGGESTED DURATION: 8 – 10 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
LG1: Students will analyze and solve geometric problems in real world context by applying postulates and understanding the geometric properties. LG2: Students will solve application based problems by extending the domain of trigonometric functions using the unit circle and modeling periodic phenomena.	
UNIT LEARNING SCALE (LG1)	
4	In addition to score 3 performances, the student can create their own geometric models for specific real world problems, and explain their reasoning for the results obtained.
3	The student can: • employ appropriate methods for when working with coordinate geometry, perimeter, area, and volume; • use theorems to model real world applications; and • analyze results to make determinations about future values.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of performances listed in score 3.
UNIT LEARNING SCALE (LG2)	
4	In addition to score 3 performances, the student can model real world events, interpret results, and determine accuracy using trigonometric functions.
3	The student can employ appropriate methods for • applying trigonometry to solve problems; • using the unit circle to evaluate trigonometric functions; • modeling periodic phenomena using trigonometric functions.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student cannot extend the domain of trigonometric functions using the unit circle and or model periodic phenomena.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: All of math, especially geometry, is based on just a few theorems and properties.	EQ1a: Why do some believe that transformations are the basis for all mathematics? EQ1b: In geometry, you use theorems and properties to prove other theorems. If you could only learn four theorems or properties, what would they be? Why?
EU2: Trigonometry helps us model non-linear scenarios, such as period phenomena.	EQ2a: Why is the study of trigonometry necessary? Where can we apply it? EQ2b: Can all periodic phenomena be modeled by trigonometric functions?
EU3: Algebra is necessary to create robust geometric models and solve otherwise unsolvable geometric problems.	EQ3: What important benefits come from “algebralizing” geometry?
EU4: Although circles are a geometric topic, their importance lies in the study of trigonometry.	EQ4: Why do we study circles? What can we do with them?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1, 2 EU2, EQ2a, b EU3, EQ3 G.SRT.11 SMP 1, 2, 3, 4 DOK 3	Students will complete a short project on the "ambiguous case" when utilizing the sine rule. Students will use a graphing program to model a triangle given specific stipulations of angles, arcs, and side lengths. Students will then have to postulate how many different triangles are possible, justify their findings, and change the input information to determine how this changes their solution.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
coordinate geometry Pythagorean theorem	Draw conclusions from the Pythagorean theorem and right angles in geometry (DOK 3) Apply concepts in geometry to the third dimension to calculate specifics and draw conclusions (DOK 3) Show how the distance formula, midpoint formula, and gradient can be used in coordinate geometry (DOK 2) Differentiate between parallel and perpendicular gradients, and how they are calculated and present themselves in a variety of ways (DOK 3)	G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
area perimeter volume	Apply measurement conversion, and accuracy of measurement in a variety of ways (DOK 3) Calculate perimeter and area (DOK 2) Apply concepts of volume, density, and capacity for application based scenarios (DOK 4)	N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
trigonometry	Utilize trigonometry ratios (DOK 1) Explain and calculate law of sines, and cosines (DOK 2) Apply concepts of area to three dimensional triangles with complex problem solving (DOK 3)	F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–*x* in terms of their values for *x*, where x is any real number. G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). G.SRT.9 Derive the formula A=½ ab sin (C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

IB MATHEMATICAL STUDIES SL, YEAR 1
UNIT 3: Logic, Sets, & Probability **SUGGESTED DURATION: 10 – 12 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
LG1: Students will use the laws of logic to analyze scenarios and make predictions. LG2: Students will use probability to analyze real world scenarios and make informed decisions.	
UNIT LEARNING SCALE (LG1)	
4	In addition to score 3 performances, the student can create their own scenarios, to model specific real world problems with sets, Venn diagrams, and logic. Students will then explain their reasoning for the results obtained.
3	The student can: • employ appropriate methods for when working with sets, Venn Diagrams, logic, probability and experimental design; and • analyze results to make determinations about future values and make predictions.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of performances listed in score 3.
UNIT LEARNING SCALE (LG2)	
4	In addition to score 3 performances, the student can critique and justify outcomes of events and make appropriate predictions.
3	The student can employ appropriate methods for • finding the probability of an event; • making predictions and informed decisions; and • justifying those predictions and or decisions.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student is not able to make predictions and informed decisions involving probability.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Using probability helps us to make inferences and predictions.	EQ1: How much would you trust probability in making life decisions?
EU2: Randomization has an effect on the conclusions formed from surveys, experiments, and observational studies.	EQ2: Can a survey, experiment, or observational study truly be random?
EU3: One can use a variety of methods to assign random variables for events in a certain sample space, then use that data to graph the corresponding probability distribution and make comparisons about expected values and actual outcomes.	EQ3: How can the same data lead to multiple conclusions?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 2 EU1, 2, 3, EQ1, 2, 3 S.CP.5 S.IC.1 SMP 1, 2, 3, 4 DOK 3	Students will use conditional probability to conduct an analysis of two events. The students will be given a scenario with two elements (i.e., colors, "what's behind the door") and determine the probability of both with random selection. They will then explain how conditional probability changes the outcomes. Students will create an additional element to the scenario they were given and justify how it would mathematically affect the probability.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
bias compound events conditional probability experimental design experimental probability population sample sampling methods tree diagrams	Investigate samples and make conjectures about the accuracy of the samples when compared to the population (DOK 3) Analyzing data to calculate probabilities in many situations (DOK 3) Explain logically how to calculate probability from compound events vs. independent events (DOK 3) Develop a logical argument for sampling; with replacement and without replacement. (DOK 3)	S.IC.2 Explain how well and why a sample represents the variable of interest from a population. Demonstrate understanding of the different kinds of sampling methods. S-CP.B.7 Apply the Addition Rule, (A or B) = $P(A) + P(B) - P(A$ and $B)$, and interpret the answer in terms of the model. S-CP.B.8 Apply the general Multiplication Rule in a uniform probability model, (A and B) = $P(A)P(B \mid A) = P(B)P(A \mid B)$, and interpret the answer in terms of the model. S.CP.1 Define a sample space and events within the sample space. Identify subsets from sample space given defined events, including unions, intersections, and complements of events. S.CP.5 Recognize and explain the concepts of independence and conditional probability in everyday situations. S.IC.3 Identify situations as either sample survey, experiment, or observational study. Discuss the appropriateness of each one's use in contexts with limiting factors. Design or evaluate sample surveys, experiments, and observational studies with randomization. Discuss the importance of randomization in these processes.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
arguments logic propositions sets truth tables Venn diagrams	Explain why Venn diagrams are utilized to describe sets within data (DOK 3) Compare data to categorized sets, and determine if the sets are disjoint (DOK 2) Analyze truth tables to determine outcomes based upon given data (DOK 3) Identify patterns in cumulative data in a variety of forms (DOK 3)	S.CP.7 Identify two events as disjoint (mutually exclusive). Calculate probabilities using the Addition Rule. Interpret the probability in context. S.IC.6 Evaluate reports based on data.

IB MATHEMATICAL STUDIES SL, YEAR 1
UNIT 4: Introduction to Mathematical Research **SUGGESTED DURATION: 2 – 4 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will investigate and research a problem in today's society and justify a solution mathematically.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can appropriately create a written proposal substantiating the need for their investigation.
3	The student can: • appropriately set up timelines for research; • choose an area of interest; • determine if research is feasible; • create topics and research questions to be used during the investigative process; • conduct research to create a proposed topic; • appropriately attain scholarly journals; • use research to substantiate or refute one's investigation; and • create a technical document outlining one's findings with conclusions.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Mathematically proficient students can define a problem and determine how to solve it.	EQ1a: What does it mean to be mathematically proficient? EQ1b: How can math be used to solve problems in today's society?
EU2: Mathematically proficient students can use mathematics to construct and justify arguments.	EQ2: What makes an argument mathematically sound?
EU3: Mathematically proficient students can explain the meaning of a problem and looking for entry points to its solution.	EQ3a: Why is it important to be able to speak math? EQ3b: How do I know what math to use to solve a problem?
EU4: Mathematically proficient students can use stated assumptions, definitions, and previously established results in constructing arguments and make and explore conjectures.	EQ4: How do I make a mathematically sound argument?
EU5: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.	EQ5: How do I know if my answer makes sense in the context of the problem?
EU6: Mathematically proficient students can notice if calculations are repeated, and look for general methods and shortcuts to mainstream a process, or utilize this information for industrial efficiency during their investigative process.	EQ6: How can you attend to detail and continually evaluate the reasonableness of the intermediate results, while working to solve a problem?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 EU1, EQ1 EU2, EQ2 EU3, EQ3 EU4, EQ4 EU5, EQ5 EU6, EQ6 SMP 1, 2, 3, 4, 5, 6, 7, 8 DOK 4	Students will complete an independent project and research investigation on a related mathematical topic of interest involving the collection of information or the generation of measurements and the analysis and evaluation of the information or measurements. Students can choose from a wide variety of project types, for example, modeling, investigations, applications and statistical surveys. Students will use mathematical methods to draw conclusions and answer questions based on their individual interests.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
beauty, power and usefulness of mathematics coherent exploration diagrams graphs ICT tools (e.g., graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software) mathematical language) mathematical processes mathematical representations (e.g., formulae, diagrams, tables, charts, graphs and models) nature of mathematics patience and persistence reflection tables technology as a tool well-organized exploration	Explain the meaning of a problem and looking for entry points to its solution (DOK 2) Analyze givens, constraints, relationships, and goals (DOK 3) Make conjectures about the form and meaning of the solution (DOK 3) Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution (DOK 3) Self-evaluate progress and make need adjustments (DOK 3) Explain correspondences between equations, verbal descriptions, tables, and graphs (DOK 2) Create diagrams and graphs of important features and relationships and determine trends (DOK 3)	Math.Practice.MP1 Make sense of problems and persevere in solving them.
	Make sense of quantities and their relationships in problem situations (DOK 2) Represent a given situation symbolically and manipulate the representing symbols without necessarily attending to their referents (DOK 3) Use quantitative reasoning to consider the units involved, attend to the meaning of quantities, and use properties (DOK 3)	Math.Practice.MP2 Reason abstractly and quantitatively.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
(same as above)	Use stated assumptions, definitions, and previously established results in constructing arguments (DOK 3) Make conjectures and build a logical progression of statements (DOK 3) Analyze situations by breaking them into cases and use counterexamples (DOK 2) Justify and communicate conclusions and respond to the arguments of others (DOK 4) Reason inductively about data, making plausible arguments that take into account the context from which the data arose (DOK 4) Compare the effectiveness of two plausible arguments (DOK 4) Distinguish correct logic or reasoning from that which is flawed and if there is a flaw explain what it is (DOK 4)	Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
	Apply mathematics to solve problems arising in everyday life, society, and the workplace (DOK 2) Identify important quantities in a practical situation and map their relationships using tools (e.g., diagrams, two-way tables, graphs, flowcharts and formulas) (DOK 2) Analyze relationships mathematically to draw conclusions (DOK 4) Interpret mathematical results in the context of the situation (DOK 3)	Math.Practice.MP4 Model with mathematics.
	Use appropriate mathematical tools (e.g., pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software) (DOK 2) Detect possible errors by strategically using estimation and other mathematical knowledge (DOK 3) Use technology to visualize the results of varying assumptions, explore consequences, and compare predictions with data (DOK 3) Identify relevant external mathematical resources (e.g., digital content located on a website) and use it to pose or solve problems (DOK 3) Use technological tools to explore and deepen understanding (DOK 2)	Math.Practice.MP5 Use appropriate tools strategically.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
(same as above)	Communicate precisely to others (DOK 2) Use clear definitions in discussion with others and in own reasoning (DOK 2) State the meaning of symbols (DOK 3) Specify units of measure and label axes to clarify the correspondence with quantities (DOK 2) Calculate accurately and efficiently (DOK 2) Express numerical answers with a degree of precision appropriate for the problem context (DOK 2)	Math.Practice.MP6 Attend to precision.
	Discern a pattern or structure (DOK 3) Shift perspectives when analyzing a problem (DOK 3)	Math.Practice.MP7 Look for and make use of structure.
	Determine repetition and look for other viable methods (DOK 3) Evaluate the reasonableness of intermediate results (DOK 3)	Math.Practice.MP8 Look for and express regularity in repeated reasoning.