FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

INTERNATIONAL BACCALAUREATE PROGRAM

MATHEMATICS SL, YEAR 2

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 29, 2016

SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING
APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS
APPENDIX B: ASSESSMENT EVIDENCE
APPENDIX C: INTERDISCIPLINARY CONNECTIONS

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IB MATHEMATICS SL, YEAR 2

COURSE PHILOSOPHY

The International Baccalaureate Organization provides the following philosophy for the teaching of mathematics and Mathematics SL: *"The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: visual artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives, with all its interdisciplinary connections, provides a clear and sufficient rationale for making the study of this subject compulsory for students studying the full diploma.*

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration."

COURSE DESCRIPTION

The International Baccalaureate Organization provides the following course description for Mathematics SL: *"The course focuses on introducing important mathematical concepts through the development of mathematical techniques. The intention is to introduce students to these concepts in a comprehensible and coherent way, rather than insisting on the mathematical rigour required for mathematics HL. Students should, wherever possible, apply the mathematical knowledge they have acquired to solve realistic problems set in an appropriate context."*

COURSE SUMMARY

COURSE GOALS

CG1: Students will model, manipulate, and reason abstractly about functions in multiple ways to develop an appreciation of the elegance and power of mathematics.
CG2: Students will analyze, model, and interpret data to communicate clearly and confidently and make sound, logical decisions.
CG3: Students will use calculus constructs to interpret and reason abstractly about quantitative models of change and deduce their consequences.
CG4: Students will solve real-world problems and produce results that are meaningful in a real-world context.

COURSE ENDURING UNDERSTANDINGS	COURSE ESSENTIAL QUESTIONS
CEU1: There are many similarities between types of functions and knowledge of one type can lead to an understanding of other types.	CEQ1a: How can our understanding of one type of function, help us to learn a new type? CEQ1b: How do we know if a feature of a function is unique to that function? CEQ1c: How can we compare functions if they are represented in different forms?
CEU2: Communication is critical to forming logical arguments that will inform decisions.	CEQ2a: How do we communicate mathematically? CEQ2b: What makes communication effective?
CEU3: Analyzing change mathematically enriches understanding of a given scenario and allows for problem solving at a high level.	CEQ3a: How is change measured mathematically? CEQ3b: What is the value of studying change in a relationship?
CEU4: Problem solving requires open-mindedness, risk-taking, and perseverance that allows one to creatively explore a variety of topics.	CEQ4a: How does risk taking relate to studying topics in mathematics? CEQ4b: Why does keeping an open-mind help students problem solve? CEQ4b: What does it mean to persevere when problem solving in mathematics?

UNIT GOALS & PACING		
UNIT TITLE	**UNIT GOALS**	**RECOMMENDED DURATION**
Unit 1: Internal Assessment	Students will explore a mathematical topic of their choosing to pursue their personal interests and demonstrate their ability to transfer their learning to a new topic.	4 weeks
Unit 2: Calculus	Students will use calculus to analyze the rates of change of a function over specific intervals.	15 weeks
Unit 3: Statistics and Probability	Students will explore, analyze and model distributions and compare these distributions to commonly studied distributions to determine the likelihood of certain outcomes.	15 weeks

IB MATHEMATICS SL, YEAR 2 **SUGGESTED DURATION:**
UNIT 1: Internal Assessment **4 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will explore a mathematical topic of their choosing to pursue their personal interests and demonstrate their ability to transfer their learning to a new topic.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can: • provide alternative methods and approaches to solving problems in the given contexts; • make connections with other topics in mathematics; • identify and correct their peers' misunderstandings; and • explain the meaning and rationale for studying these topics.
3	The student can: • communicate mathematical ideas appropriately and coherently in a way that is logically developed and easy to follow; • demonstrate genuine personal engagement with the topic of their choosing; • reflect upon their work in a meaningful manner that shows growth of understanding throughout the exploration process; and • demonstrate an understanding of mathematics that is commensurate of the level of the course.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Being able to explore mathematics will lead to a richer understanding of the mathematics and a better appreciation of the content.	EQ1a: What are the different ways to explore mathematics? How do I know which are appropriate for a given topic? EQ1b: How can I go beyond research and interact with the mathematics on a more personal level?
EU2: Writing mathematical analyses will improve communication skills and will therefore lead to a higher level of mathematical competence.	EQ2a: What are the similarities and differences between communicating and writing mathematically versus another subject? EQ2b: Why is it valuable to be able to communicate about mathematics?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU1, EU2 EQ1a, b, EQ2a, b SMP 1-8 DOK 4	The common assessment for this unit is the Internal Assessment established by IB. Students will complete an independent exploration project and research investigation on a related mathematical topic of interest involving the collection of information or the generation of measurements and the analysis and evaluation of the information. Students can choose from a wide variety of project types, such as modeling, investigations, applications, statistical analysis or a combination of multiple of these. Students will use mathematical methods to draw conclusions and answer questions based on their individual interests. Each project must contain: • a statement of the task and plan; • research, data, and/or mathematical exploration that have been collected and/or generated; • an analysis of the research, data and/or mathematical exploration; • proof of personal engagement to the exploration; • reflection of exploration; • appropriate notation and terminology.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
appropriate mathematical language (e.g., notation, symbols, terminology) beauty, power and usefulness of mathematics coherent exploration graphs, tables, and diagrams mathematical processes nature of mathematics patience and persistence power of technology as a mathematical tool well-organized exploration	Explain the meaning of a problem and looking for entry points to its solution (DOK2) Analyze givens, constraints, relationships, and goals (DOK3) Make conjectures about the form and meaning of the solution and plan a solution pathway (DOK3) Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution (DOK3) Monitor and evaluate their progress and change course if necessary (DOK3) Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends (DOK3) Understand, analyze, and critique the approaches of others to solving complex problems and identify correspondences between different approaches. (DOK3)	Math.Practice.MP1 Make sense of problems and persevere in solving them.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
ICT tools (i.e., graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software) mathematical language) mathematical representation (e.g., formulae, diagrams, tables, charts, graphs and models reflection	Make sense of quantities and their relationships in problem situations (DOK2) Decontextualize-to abstract a given situation, represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents (DOK3) Contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved (DOK3) Use quantitative reasoning to consider the units involved, attend to the meaning of quantities, and use different properties of operations and objects (DOK3)	Math.Practice.MP2 Reason abstractly and quantitatively.
	Use stated assumptions, definitions, and previously established results in constructing arguments (DOK3) Make conjectures and build a logical progression of statements to explore the truth of their conjectures (DOK3) Analyze situations by breaking them into cases, and can recognize and use counterexamples (DOK2) Justify conclusions, communicate them to others, and respond to the arguments of others (DOK 4) Reason inductively about data, making plausible arguments that take into account the context from which the data arose (DOK4) Compare the effectiveness of two plausible arguments (DOK4) Distinguish correct logic or reasoning from that which is flawed and if there is a flaw explain what it is (DOK4)	Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
	Apply mathematics solve problems arising in everyday life, society, and the workplace (DOK2) Identify important quantities in a practical situation and map their relationships using tools (e.g., diagrams, two-way tables, graphs, flowcharts and formulas) (DOK2) Analyze relationships mathematically to draw conclusions (DOK4) Interpret mathematical results in the context of the situation and reflect on whether the results make sense (DOK3)	Math.Practice.MP4 Model with mathematics.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
(same as above)	Use appropriate mathematical tools (e.g., pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software) (DOK2) Detect possible errors by strategically using estimation and other mathematical knowledge (DOK3) Use technology to visualize the results of varying assumptions, explore consequences, and compare predictions with data (DOK3) Identify relevant external mathematical resources (e.g., digital content located on a website) and use it to pose or solve problems (DOK3) Use technological tools to explore and deepen understanding (DOK2)	Math.Practice.MP5 Use appropriate tools strategically.
	Communicate precisely to others (DOK2) Use clear definitions in discussion with others and in own reasoning (DOK2) State the meaning of symbols (DOK3) Specify units of measure, and label axes to clarify the correspondence with quantities in a problem (DOK2) Calculate accurately and efficiently (DOK2) Express numerical answers with a degree of precision appropriate for the problem context(DOK2)	Math.Practice.MP6 Attend to precision.
	Discern a pattern or structure (DOK3) Shift perspectives when analyzing a problem (DOK3)	Math.Practice.MP7 Look for and make use of structure.
	Determine repetition and look for other viable methods (DOK3) Evaluate the reasonableness of intermediate results (DOK3)	Math.Practice.MP8 Look for and express regularity in repeated reasoning.

IB MATHEMATICS SL, YEAR 2 **SUGGESTED DURATION:**
UNIT 2: Calculus **15 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will use calculus to analyze the rates of change of a function over specific intervals.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can: • provide alternative methods and approaches to solving problems in the given contexts; • make connections with other topics in mathematics; • identify and correct their peers' misunderstandings; and • explain the meaning and rationale for studying these topics.
3	Students can: • find derivatives and integrals for a variety of functions and explain the conceptual understanding of completing such a task; • represent derivatives and integrals in a variety of manners (e.g., algebraically, graphically, and verbally); and • apply the properties of finding derivatives and integrals to a variety of application problems and explain the meaning of the solutions in context.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Derivatives and integrals can be used to solve a variety of problems related to instantaneous rate of change from a variety of contexts.	EQ1a: How do I know when exploring a problem whether I should find the derivative or integral? EQ1b: How do derivatives and integrals lead to a better understanding of the data that a function describes?
EU2: Differentiation and definite integration are inverse operations.	EQ2a: What similarities and differences exist in finding derivatives and integrals for different types of function? EQ2b: How can an understanding of this inverse relationship help to find one given the other?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU1 & EQ1b M.HS.F.LE.A.3 M.HS.F.BF.A.1 M.HS.F.IF.B.4, 5, 6 M.HS.F.IF.C.7, 8, 9 SMP 1-8 DOK 4	Students will be asked to explore the relationship between position, velocity and acceleration of an object in motion of their choosing. They can select something small that moves rather fast, something large that moves more slowly or anything in between. Because the data will be experimental, regression techniques from statistics will be required to find the model that best fits the function of position versus time. Once this is found, students will explore the velocity and acceleration models and use it to learn more about this object in motion. Students will create their own functions, graphs and give explanations of each giving clear evidence of conceptual understanding of derivatives and integrals of each.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
chain rule continuity convergence derivative derivative from first principles derivatives of sec x, csc x, cot x, a^x, $\log_a x$, arcsin x, arccos x, and arctan x derivatives of x^n, sin x, cos x, tan x, e^x and ln x differentiation equations of tangents and normal gradient function higher derivatives implicit differentiation increasing/decreasing functions limit local maximum and minimum values optimization points of inflection with zero and non-zero gradients product rules quotient rules related rates of change the relationship between the graphs of f, f', and f'' the second derivative	Find and interpret the limit at a specific value for a variety of functions (DOK3) Relate limits to derivatives in concept and for a variety of functions (DOK2) Explore functions as they change over the domain and determine the meaning for the derivative at the different points (DOK3) Investigate the relationship between a function, its derivative, and higher derivatives to better understand the nature of the function (DOK3) Differentiate between different differentiation rules for a variety of functions (DOK3)	F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. F-BF.A.1 Write a function that describes a relationship between two quantities. F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
acceleration *a,* anti-differentiation with a boundary condition displacement *s*, indefinite integral of x^n, sin x, cos x, $\frac{1}{x}$ and e^x indefinite integration as anti-differentiation integral integration by inspection, or substitution kinematic problems total distance traveled velocity *v* volumes of revolution about the x-axis	Find and interpret integrals for a variety of functions (DOK3) Explore the relationship between derivatives and integrals for a variety of functions (DOK3) Distinguish between different integration techniques depending on the type of integration problem that a function presents (DOK2) Apply integral calculus to a variety of kinematic problems (DOK3)	F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

IB MATHEMATICS SL, YEAR 2 **SUGGESTED DURATION:**
UNIT 3: Statistics and Probability **15 WEEKS**

UNIT OVERVIEW

UNIT LEARNING GOALS

Students will explore, analyze and model distributions and compare these distributions to commonly studied distributions to determine the likelihood of certain outcomes.

UNIT LEARNING SCALE

4	In addition to score 3 performances, the student can: • provide alternative methods and approaches to solving problems in the given contexts; • make connections with other topics in mathematics; • identify and correct their peers' misunderstandings; and • explain the meaning and rationale for studying these topics.
3	The student can: • analyze distributions of data using appropriate measures and graphical displays; • appropriately model the data and discuss the strength of the relationship based on the model type selected; and • use probability to discuss the likelihood of specific outcomes based on experimental data and known distributions of theoretical data.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Sample data will vary depending on many factors including sample method, sample size, and bias.	EQ1a: What measures can be taken to limit the variability within a sample? EQ1b: How does this variability affect our decision making process?
EU2: Making a prediction is a multi-step process that requires the continual analysis and summary of data in order to find the patterns within the distribution. Some data distributions will yield predictions that will be more accurate than others.	EQ2a: How do you know when to use which measures of numerical summary for different distributions? EQ2b: What makes one model better than another when trying to find patterns within a distribution? EQ2c: How can you tell how accurate a prediction is going to be?
EU3: Probability leads to a better understanding of distributions, which can be used to make predictions and inferential decisions about populations.	EQ3a: How is the probability of certain outcomes dependent on the distribution of data? EQ3b: Should data from a sample be used to make inferential decisions about a population?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU1, EU2 EQ1a, b, EQ2a, b, c M.HS.S.IC.A.1 M.HS.S.ID.A.2, 3, 4 M.HS.S.ID.B.6 M.HS.S.ID.C.9 SMP 1-8 DOK 2-4	Students will create a statistical question that can be answered based on an experiment or observational study. Students will outline the design of the study and identify any lurking variables or other limitations prior to data collection. Once data is collected, students will analyze the distribution and summarize numerically and graphically. Probability will then be used to discuss the likelihood of specific outcomes. The students will answer the initial question based on their analysis. Their answer will demonstrate a clear understanding of data production, data analysis, probability, and inferential statistics. Students will justify the strength of their study in the context of statistics and offer methods to improve their study.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
boxplots discrete and continuous data frequency distributions frequency histograms interval width mid-interval values modal class outliers population sample upper and lower interval boundaries	Investigate samples and make conjectures about the accuracy of the sample when compared to the population (DOK3) Represent data with plots on the real number line, using appropriate display type (DOK2) Explain logically and mathematically patterns and deviations from these patterns in data (DOK3) Develop a logical argument for dividing data using appropriate means, which will vary depending on the data that is being analyzed (DOK3)	S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
central tendency cumulative frequency dispersion IQR quartiles, percentiles range standard deviation statistical measures variance	Explain why certain statistical measures are more appropriate than others for a variety of distributions and interpret these values (DOK3) Compare distributions using appropriate measures of center and spread (DOK2) Analyze the effect of changes to a data set by changing units (DOK3) Identify patterns in cumulative data in a variety of forms (DOK3)	S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
linear correlation of bivariate data Pearson's product-moment correlation coefficient *r* lines of best fit regression line of y on x scatter diagrams	Represent bivariate data in a variety of forms including table, graphical and equation (DOK3) Assess the strength of a linear relationship using a variety of methods and synthesize these methods to determine the appropriateness of the model (DOK3) Interpret key features of a linear model in context and comment on the values using logical arguments (DOK3) Explain the differences between correlation and causation when it refers to relationships for bivariate data (DOK2)	S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. S-ID.B.6b Informally assess the fit of a function by plotting and analyzing residuals. S-ID.B.6c Fit a linear function for a scatter plot that suggests a linear association. S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S-ID.C.9 Distinguish between correlation and causation.
normal curves normal distributions standardization of normal variables (z-values, z-scores)	Convert data to standardized values to compare values with different properties (DOK2) Cite evidence to determine that a distribution is normal (DOK3) Find probabilities for distributions that are normal and interpret these values (DOK2)	S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
binomial distribution discrete data discrete random variables probability distribution expected value mean variance	Differentiate between discrete and continuous random variables in concept, representation and probability (DOK2) Represent discrete random variables and their probabilities in a probability distribution and a probability histogram (DOK2) Explain the requirements for a discrete random variable to follow a binomial distribution and classify examples as binomial or not (DOK3) Find and interpret the expected value, variance and standard deviation for discrete random variables (DOK2)	S-MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S-MD.A.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S-MD.A.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S-MD.A.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. S-MD.B.5a Find the expected payoff for a game of chance. S-MD.B.5b Evaluate and compare strategies on the basis of expected values.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
combined events complement conditional probability equally likely outcomes independent events mutually exclusive events outcome probabilities with and without repetition probability sample space and event tables of outcomes tree diagrams trial Venn diagrams	Develop models to find the probability of a variety of events using both experimental and theoretical methods and compare the accuracy of these methods (DOK3) Find and interpret the probability of combined events using appropriate notation and theoretical approaches (DOK3) Develop a logical argument for why combined events are mutually exclusive (disjoint) and represent this in a variety of manners (DOK3) Determine whether events are independent using empirical knowledge and interpret what it means for events to be independent (DOK3)	S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). S-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S-CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S-CP.B.6 Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model. S-CP.B.7 Apply the Addition Rule, (*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model. S-CP.B.8 Apply the general Multiplication Rule in a uniform probability model, (*A* and *B*) = *P*(*A*)*P*(*B*|*A*) = *P*(*B*)*P*(*A*|*B*), and interpret the answer in terms of the model. S-CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.