FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

INTERNATIONAL BACCALAUREATE PROGRAM

MATHEMATICS HL, YEAR 1

Grade Level: 11

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 29, 2016

SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING
APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS
APPENDIX B: ASSESSMENT EVIDENCE
APPENDIX C: INTERDISCIPLINARY CONNECTIONS

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

Curriculum Writing Committee

Mr. David Patterson

Supervisor

Ms. Mary Hough

IB MATHEMATICS HL, YEAR 1
COURSE PHILOSOPHY
The International Baccalaureate Organization provides the following philosophy for the teaching of mathematics and Mathematics HL: *"The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: visual artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives, with all its interdisciplinary connections, provides a clear and sufficient rationale for making the study of this subject compulsory for students studying the full diploma.* *This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems."*
COURSE DESCRIPTION
The International Baccalaureate Organization provides the following description for the teaching of mathematics and Mathematics HL*: The course focuses on developing important mathematical concepts in a comprehensible, coherent and rigorous way. This is achieved by means of a carefully balanced approach. Students are encouraged to apply their mathematical knowledge to solve problems set in a variety of meaningful contexts. Development of each topic should feature justification and proof of results. Students embarking on this course should expect to develop insight into mathematical form and structure, and should be intellectually equipped to appreciate the links between concepts in different topic areas. They should also be encouraged to develop the skills needed to continue their mathematical growth in other learning environments. . . This course is a demanding one, requiring students to study a broad range of mathematical topics through a number of different approaches and to varying degrees of depth."*

COURSE SUMMARY	
COURSE GOALS	
CG1: Students will model, manipulate, and develop abstract reasoning skills about different types of functions while making mathematical connections and building an appreciation for the elegance and power of mathematics. CG2: Students will analyze, model, and interpret data to communicate clearly and confidently and make sound, logical decisions based on probability models. CG3: Students will use calculus constructs to interpret and reason abstractly about quantitative models of change and deduce their consequences. CG4: Students will create, analyze, and solve real-world problems and communicate results that are meaningful in a variety of real-world contexts.	
COURSE ENDURING UNDERSTANDINGS	COURSE ESSENTIAL QUESTIONS
CEU1: There are many similarities between types of functions and knowledge of one type can lead to an understanding of other types.	CEQ1a: How can our understanding of one type of function, help us to learn a new type? CEQ1b: How do we know if a feature of a function is unique to that function? CEQ1c: How can we compare functions if they are represented in different forms?
CEU2: Communication is critical to forming logical arguments that will inform decisions.	CEQ2a: How do we communicate mathematically? CEQ2b: What makes communication effective?

UNIT GOALS & PACING		
UNIT TITLE	UNIT GOALS	RECOMMENDED DURATION
Unit 1: Algebra	Students will model, manipulate, and reason abstractly using algebra in multiple ways to develop an appreciation of the elegance and power of mathematics.	6 weeks
Unit 2: Functions and Equations	Students will explore the notion of a function as a unifying theme in mathematics and apply functional methods to model, manipulate, and reason abstractly in a variety of mathematical situations.	6 weeks
Unit 3: Circular Functions and Trigonometry	Students will model, manipulate, and reason abstractly about circular functions in multiple ways to solve problems involving trigonometry and explain real world applications.	8 weeks
Unit 4: Vectors	Students will model, manipulate, and reason abstractly about objects and forces in action using vectors.	4 weeks
Unit 5: Calculus	Students will use calculus to analyze the rates of change of a function over specific intervals.	10 weeks

IB MATHEMATICS HL, YEAR 1 UNIT 1: ALGEBRA	SUGGESTED DURATION: 6 WEEKS

<table>
<tr><th colspan="3">UNIT OVERVIEW</th></tr>
<tr><td colspan="3">UNIT LEARNING GOALS</td></tr>
<tr><td colspan="3">Students will model, manipulate, and reason abstractly using algebra in multiple ways to develop an appreciation of the elegance and power of mathematics.</td></tr>
<tr><td colspan="3">UNIT LEARNING SCALE</td></tr>
<tr><td>4</td><td colspan="2">In addition to score 3 performances, the student can:
• provide alternative methods and approaches to solving problems in the given contexts;
• make connections with other topics in mathematics;
• identify and correct their peers’ misunderstandings; and
• explain the meaning and rationale for studying these topics.</td></tr>
<tr><td>3</td><td colspan="2">The student can:
• model in a variety of ways and reason abstractly for both arithmetic and geometric sequences and series in theoretical and application scenarios;
• explore relationships between, manipulate and reason abstractly for exponential and logarithmic functions; and
• explore Pascal’s triangle and use it to model, manipulate and reason abstractly for binomial expansion situations.</td></tr>
<tr><td>2</td><td colspan="2">The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.</td></tr>
<tr><td>1</td><td colspan="2">The student needs assistance to avoid major errors in attempting to reach score 3 performances.</td></tr>
<tr><td>0</td><td colspan="2">Even with help, the student does not exhibit understanding of modeling, manipulating and reasoning abstractly for these topics.</td></tr>
<tr><td colspan="2">ENDURING UNDERSTANDINGS</td><td>ESSENTIAL QUESTIONS</td></tr>
<tr><td colspan="2">EU1: Pattern-seeking in mathematics allows us to better understand the nature of a relationship and allow us to make hypotheses and predictions.</td><td>EQ1a: Can all mathematical relationships be modeled in a way that is meaningful? If not, how can you tell if a relationship is meaningful?
EQ1b: What evidence allows us to be confident in making hypotheses and predictions based on patterns?</td></tr>
<tr><td colspan="2">EU2: All mathematical operations can be undone or reversed through another mathematical operation.</td><td>EQ2a: If you know what operations “undo” each other, how does this help when working with mathematical equations?
EQ2b: How does knowing the relationship between inverse operations affect our understanding of the graphical representation of two functions?</td></tr>
</table>

<table>
<tr><th colspan="2">COMMON ASSESSMENT</th></tr>
<tr><td>ALIGNMENT</td><td>DESCRIPTION</td></tr>
<tr><td>LG1
EU1, EQ1a, 1b
F.BF.A.1.A
F.LE.A.2, 3
SMP 1-8
DOK 2-4</td><td>Students will explore two separate job offers for a recent college graduate. Each job will offer the same starting salary ($60,000), but one job’s salary will grow arithmetically ($3500 raise per year) while the other will grow geometrically (3% raise per year). Students will begin by making a hypothesis about which job is the better offer and justifying their hypothesis mathematically. Students will then create mathematical models of the two scenarios. Based on their models, students will write a justification for the offer they deem best. Students must explain their choice based on how long they expect to be working at this job. Students must also answer the following questions: (1) Would your decision change if you were 62 instead of 22? (2) Would your decision change if the starting salary was $90,000 instead?</td></tr>
</table>

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
arithmetic sequences arithmetic series geometric sequences geometric series sigma notation sum of a finite arithmetic series sum of finite geometric series sum of infinite geometric series	Generate and display sequences in several ways, including explicit and recursive functions (DOK3) Find the sums of finite arithmetic and finite and infinite geometric series (DOK2)	A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems
		F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
		F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
		F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
change of base exponents laws of exponents laws of logarithms logarithms	Simplify and solve a variety of exponential and logarithmic expressions and equations (DOK2) Represent exponential and logarithmic functions in a variety of ways (DOK2) Explain the relationship between exponential functions and logarithmic functions (DOK3)	F-IF.C.8b Use the properties of exponents to interpret expressions for exponential functions.
		A-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions.
binomial expansion binomial theorem combinations counting principle permutations	Explain the differences between permutations and combinations (DOK1) Apply permutations and combinations to real world problems (DOK3) Finding permutations and combination using both the formula and technology (DOK2) Expand binomials through a variety of means demonstrating an understanding of the binomial theorem and Pascal's Triangle (DOK3)	S-CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.
		A-APR.C.5 Know and apply the binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and y for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.
proof by mathematical induction	Use proof by induction to justify something using general terms (DOK3)	MP7 Look for and make use of structure.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
a + bi argument Cartesian form *z = a + ib* complex numbers complex plane conjugates de Moivre's theorem fundamental theorem of algebra modulus modulus-argument (polar) form *n*th roots of a complex number	Find all roots of a polynomial using various methods(DOK2) Use roots of a polynomial to write and graph a polynomial (DOK2) Use sums, products and quotients to find and prove the zeros exist (DOK3) Convert between forms (DOK1) Explain the relationship between the complex plane and a coordinate plane (DOK3) Explain the value of working with a complex plane (DOK3) Use de Moivre's theorem to find the nth root of a complex number (DOK2) Prove de Moivre's theorem using mathematical induction (DOK3) Find all roots of a polynomial given a variety of conditions (DOK2) Explain the relationship between the degree of a polynomial and the number of roots using the Fundamental Theorem of Algebra (DOK3)	A-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*. A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
systems of linear equations	Solve systems of equations (DOK3)	A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

IB MATHEMATICS HL, YEAR 1 UNIT 2: FUNCTIONS AND EQUATIONS	SUGGESTED DURATION: 6 WEEKS

<table>
<tr><th colspan="3">UNIT OVERVIEW</th></tr>
<tr><td colspan="3">UNIT LEARNING GOALS</td></tr>
<tr><td colspan="3">Students will explore the notion of a function as a unifying theme in mathematics and apply functional methods to model, manipulate, and reason abstractly in a variety of mathematical situations.</td></tr>
<tr><td colspan="3">UNIT LEARNING SCALE</td></tr>
<tr><td>4</td><td colspan="2">In addition to score 3 performances, the student can:
• provide alternative methods and approaches to solving problems in the given contexts;
• make connections with other topics in mathematics;
• identify and correct their peers’ misunderstandings; and
• explain the meaning and rationale for studying these topics.</td></tr>
<tr><td>3</td><td colspan="2">The student can:
• model, manipulate and reason abstractly for a variety of functions;
• hypothesize about new functions based on what is known about functions previously studied; and
• explain the similarities and differences between each type of function and make conjectures about why these relationships exist.</td></tr>
<tr><td>2</td><td colspan="2">The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.</td></tr>
<tr><td>1</td><td colspan="2">The student needs assistance to avoid major errors in attempting to reach score 3 performances.</td></tr>
<tr><td>0</td><td colspan="2">Even with help, the student does not exhibit understanding of modeling, manipulating, reasoning abstractly, and making connections for these topics.</td></tr>
<tr><td colspan="2">ENDURING UNDERSTANDINGS</td><td>ESSENTIAL QUESTIONS</td></tr>
<tr><td colspan="2">EU1: Functions can be represented in multiple forms that can be explored and manipulated in ways that are powerful and meaningful.</td><td>EQ1a: Is one form of a representation more useful than another to make understanding more meaningful? How do we know?
EQ1b: How can functions be manipulated to better understand the nature of the relationship?</td></tr>
<tr><td colspan="2">EU2: Function analysis is the basis for exploration, representation and interpretation of many mathematical topics.</td><td>EQ2a: What is it about functions that make them the basis of other mathematical topics?
EQ2b: If I understand how one function transforms, how can I hypothesize about the transformation of a new type of function?
EQ2c: Why are the domain and range critical to furthering our understanding of a function?</td></tr>
</table>

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU1, EU2, EQ1a, b, EQ2a, c F.IF.A.3, C.8 A.APR.D.6 A.CED.A.3, 4 SMP 1-8 DOK 2, 3, 4	Students will collect bivariate data on a topic of their choosing. Using the data, they will identify the type of function. Data may or may not fit exactly onto a curve, but students should identify the function type by exploring the graph and common features of the data. Students will then create a mathematical model of the function, identify the key features, and explain the transformations from the parent graph and what they imply about the function. Students will also have to explain the strengths and weaknesses of the model. Students will then find the inverse of this function and explain the value of exploring the inverse.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
composite functions domain domain restriction identity function image (value) odd and even functions one-to-many functions one-to-one functions range self-inverse functions	Find and interpret the domain and range of a variety of functions (DOK3) Identify the key properties of a variety of functions (DOK1) Compose functions and use these compositions to determine if two functions are inverses (DOK2) Find inverse functions (DOK3)	F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then (*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = (*x*).
		F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
		F-IF.B.5 Relate the domains a function to its graph and, where applicable, to the quantitative relationship it describes.
		F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
		F-BF.B.4b Verify by composition that one function is the inverse of another.
domain and range graph of y = |f(x)| graph of y = f(|x|) horizontal asymptotes intercepts maximum and minimum values symmetry vertical asymptotes y = f(x)	Identify and interpret key features of a function given its equation, graph, or description (DOK2) Model functions in a variety of ways (DOK2) Identifying, interpreting, graphing and writing functions involving absolute values and reciprocals (DOK3)	F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
		F-IF.C.8a - Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
translations stretches reflections graph of the inverse function as a reflection in y = x	Identifying various transformations for a variety of functions graphically and algebraically (DOK2) Convert between different forms of a function to identify the transformations (DOK2)	F-BF.B.4c Read values of an inverse function from a graph or a table, given that the function has an inverse. G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
discriminant factor theorem fundamental theorem of algebra graphical or algebraic models for polynomials up to degree 3 polynomial functions quadratic formula remainder theorem roots solution of $a^x = b$ using logarithms solutions of g(x) > f(x) sum and product of roots of polynomial equations	Solve quadratic and higher degree polynomials using a variety of methods and interpret the meaning of these solutions (DOK2) Investigate the nature of roots using the factor theorem, remainder theorem, sum and product of roots theorem and fundamental theorem of algebra (DOK2) Differentiate when to use different theorems to find roots (DOK3) Determine if values are roots and find remainders (DOK2) Compare functions algebraically and graphically to find solutions and values of the domain (DOK3) Use technology to solve a variety of equations including those where there is no appropriate analytic approach (DOK 2)	A-APR.B.2 Know and apply the Remainder Theorem: For a polynomial (x) and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A-REI.B.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. A-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
exponential functions logarithmic functions rational functions	Find key features of rational, exponential and logarithmic functions (DOK2) Prove that exponential functions and logarithmic functions are inverses (DOK3)	F-IF.C.7e Graph exponential and *logarithmic* functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
		F-BF.B.5 Understand the inverse relationship between exponents and *logarithms* and use this relationship to solve problems involving logarithms and exponents.
		F-LE.A.4 For exponential models, express as a logarithms the solution to *ab* to the *ct* power = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.
		F-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
		F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

IB MATHEMATICS HL, YEAR 1 UNIT 3: CIRCULAR FUNCTIONS AND TRIGONOMETRY	SUGGESTED DURATION: 8 WEEKS

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will model, manipulate, and reason abstractly about circular functions in multiple ways to solve problems involving trigonometry and explain real world applications.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can: • provide alternative methods and approaches to solving problems in the given contexts; • make connections with other topics in mathematics; • identify and correct their peers' misunderstandings; and • explain the meaning and rationale for studying these topics.
3	The student can: • model, manipulate, and reason abstractly for circular functions; • model, manipulate, and reason abstractly for trigonometric problems; and • explain the meaning of circular functions and trigonometric problems in a real-world scenario.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of modeling, manipulating, reasoning abstractly and explaining the meaning for these topics.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: The periodic nature of trigonometric functions is based in their relationship with the circular functions from which they are defined.	EQ1: What does the repetitive or periodic nature of a trigonometric function have to do with understanding the critical information about the function?
EU2: Manipulation of trigonometric functions exposes equivalent ways to represent functions and is critical to the understanding of various functions.	EQ2a: What does it mean that two different trigonometric functions are equivalent? EQ2b: What are advantages and disadvantages of using the different methods for determining if trigonometric functions are equivalent?
EU3: The solutions to a trigonometric equation over a finite interval represent only a small portion of the total number of solutions.	EQ3: If one solution of a trigonometric equation is known, how can more solutions be found? How can the solution be generalized?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG1 EU1, EQ1 EU2, EQ2a, 2b F.TF.A.4 F.TF.B.5, 6, 7 SMP 1-8 DOK 2-3	Students will explore tidal data from a geographical location of their choosing. Using the data, they will describe key features about the relationship between time and sea level, graph it, and model it using a sinusoidal function. Students will use this model to make predictions about sea level heights for different times. Finally, students will explore other sinusoidal functions. For instance, they could research tides for this location at other times (specifically spring and neap tides), describe the transformations at these times and discuss the meaning of these transformations.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
area of a sector circle length of an arc radian measure of angles unit circle and special angles	Convert fluently between different units of measure for angles (DOK1) Investigate the relationship between a central angle, the arc intercepted, and the area of the sector created (DOK3) Explore relationships between the unit circle and a rectangular graph to realize the periodic nature of trigonometry (DOK3)	F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
compound angle identities $\cos\theta$ double angle identities Pythagorean identities reciprocal trigonometric ratios $\sec\theta$, $\csc\theta$, and $\cot\theta$ $\sin\theta$ $\tan\theta$ values of sin, cos, and tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples	Find trigonometric ratios using the unit circle and special right triangles (DOK2) Explain the relationship between the cosine, sine and tangent of an angle using the Pythagorean theorem (DOK3) Define the relationship between trigonometric ratios and their reciprocals (DOK3) Use Pythagorean and compound angle identities to solve trigonometric equations and explain these solutions from algebraic and geometric perspectives (DOK3) Solve trigonometric equations in a finite interval, including the use of trigonometric identities and factorization (DOK 3)	F-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the *unit* circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. F-TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. F-TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
area of a triangle as $\frac{1}{2}ab\sin C$ cosine rule sine rule including the ambiguous case	Use cosine and sine rule to solve trigonometric equations including the ambiguous case for law of sines (DOK3) Use trigonometry to find the area of a non-right triangle (DOK2)	G-SRT.D.10 Prove the Laws of Sines and cosines and use them to solve problems. G-SRT.D.11 Understand and apply the Law of Sines and the Law of cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
composite functions of the form f(x) = a sin(b(x + c)) + d domains of inverse functions domains of inverse functions inverse functions f(x) = arcsin x, arccos x, arctan x	Algebraically, compose functions to identify transformations for trigonometric functions (DOK2) Explore the relationship between the domain and range of a trig function and its inverse (DOK3)	G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). F-BF.B.4b Verify by composition that one function is the inverse of another.

IB MATHEMATICS HL, YEAR 1 UNIT 4: VECTORS	SUGGESTED DURATION: 4 WEEKS

UNIT OVERVIEW

UNIT LEARNING GOALS

Students will model, manipulate, and reason abstractly about objects and forces in action using vectors.

UNIT LEARNING SCALE

4	In addition to score 3 performances, the student can: • provide alternative methods and approaches to solving problems in the given contexts; • make connections with other topics in mathematics; • identify and correct their peers' misunderstandings; and • explain the meaning and rationale for studying these topics.
3	In a variety of situations, the student can explore algebraic and geometric approaches to vectors in order to model, manipulate and reason abstractly about objects and forces in nature.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with help, the student does not exhibit understanding of modeling, manipulating and reasoning abstractly for these topics.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Vectors represent an approach to model, manipulate, and interpret objects and forces in action in a way that is both practical and meaningful.	EQ1: What are the benefits of abstractly representing objects and forces in action?
EU2: Combining algebraic and geometric approaches to vectors produces a more well-rounded understanding on the topic.	EQ2: How can you determine for which situations an algebraic understanding of vectors is more beneficial than a geometric understanding (and vice versa)?

COMMON ASSESSMENT

ALIGNMENT	DESCRIPTION
LG1 EU1, EQ1 N.VM.A.2, 3 N.VM.B.4, 5 SMP 1-8 DOK 2-3	Students will choose an object in motion to explore (e.g., boats in water, planes in the sky, swimmers in water). Each object will be in motion from a particular location at a specific velocity, selected by the student. They will first find the position of the vectors of each object and explain the meaning of the vectors. They will then find the distances traveled and how far apart the objects are at various times. Students will explore addition of vectors by exploring the impact of another force (e.g., water current, wind resistance).

TARGETED STANDARDS

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
AB = b – a Cartesian equation of a plane coincident lines intersecting lines intersections and angles in/with a plane kinematics multiplication by a scalar, kv normal vector / rn = an parallel lines perpendicular/parallel vectors points of intersection position vectors properties of the vector product geometric interpretation of |v x w| scalar product skew lines the angle between two vectors the zero vector 0 unit/base vectors i, j, k vector equation in two and three dimensions vector equation of a plane vectors	Identify the magnitude and direction of vectors (DOK1) Use appropriate symbols for vectors and their magnitudes (DOK1) Represent vectors as a directed line segment (DOK2) Perform arithmetic operations on vectors (DOK2)	N-VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes *(e.g., **v**, |**v**|, ||**v**)*
	Find the components of a vector by subtracting the coordinates of the initial point from the terminal point (DOK2) Describe the transformations of vectors (DOK2)	N-VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
	Solve problems involving velocity (DOK1) Solve problems involving quantities that can be represented by vectors (DOK2)	N-VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.
	Determine the magnitude and direction of the sum of two vectors given the magnitude and direction of each (DOK2) Add vectors using various methods (DOK2)	N-VM.B.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
		N-VM.B.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
	Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise (DOK2)	N-VM.B.4c Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	Represent scalar multiplication of a vector graphically (DOK2) Perform scalar multiplication of a vector component-wise (DOK2)	N-VM.B.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as (v_x, *v* subscript *y*) = (cv_x, *cv* subscript *y*).
	Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v (DOK2) Compute the direction of cv knowing that when |c|v is not 0, the direction of cv is either along v or against v (DOK2)	N-VM.B.5b Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|***v***. Compute the direction of *c**v*** knowing that when |*c*|***v*** ≠ 0, the direction of *c**v*** is either along ***v*** (for $c > 0$) or against ***v*** (for $c < 0$).

IB MATHEMATICS HL, YEAR 1 UNIT 5: CALCULUS	SUGGESTED DURATION: 10 WEEKS

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will use calculus to analyze the rates of change of a function over specific intervals.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can: • provide alternative methods and approaches to solving problems in the given contexts; • make connections with other topics in mathematics; • identify and correct their peers' misunderstandings; and • explain the meaning and rationale for studying these topics.
3	Students can: • find derivatives and integrals for a variety of functions and explain the conceptual understanding of completing such a task; • represent derivatives and integrals in a variety of manners (e.g., algebraically, graphically, and verbally); and • apply the properties of finding derivatives and integrals to a variety of application problems and explain the meaning of the solutions in context.
2	The student sometimes needs assistance from a teacher, makes minor mistakes, and/or can do the majority of level 3 performances.
1	The student needs assistance to avoid major errors in attempting to reach score 3 performances.
0	Even with assistance, the student does not exhibit understanding of the performances listed in score 3.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Derivatives and integrals can be used to solve a variety of problems related to instantaneous rate of change from a variety of contexts.	EQ1a: How do I know when exploring a problem whether I should find the derivative or integral? EQ1b: How do derivatives and integrals lead to a better understanding of the data that a function describes?
EU2: Differentiation and definite integration are inverse operations.	EQ2a: What similarities and differences exist in finding derivatives and integrals for different types of function? EQ2b: How can an understanding of this inverse relationship help to find one given the other?

COMMON ASSESSMENT	
ALIGNMENT	**DESCRIPTION**
LG1 EU1 & EQ1b F.LE.A.3 F.BF.A.1 F.IF.B.4, 5, 6 F.IF.C.7, 8, 9 SMP 1-8 DOK 4	Students will be asked to explore the relationship between position, velocity and acceleration of an object in motion of their choosing. They can select something small that moves rather fast, something large that moves more slowly or anything in between. Because the data will be experimental, regression techniques from statistics will be required to find the model that best fits the function of position versus time. Once this is found, students will explore the velocity and acceleration models and use it to learn more about this object in motion. Students will create their own functions, graphs and give explanations of each giving clear evidence of conceptual understanding of derivatives and integrals of each.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
chain rule continuity convergence derivative derivative from first principles derivatives of sec x, csc x, cot x, a^x, $\log_a x$, arcsin x, arccos x, and arctan x derivatives of x^n, sin x, cos x, tan x, e^x and ln x differentiation equations of tangents and normal gradient function higher derivatives implicit differentiation increasing/decreasing functions limit local maximum and minimum values optimization points of inflection with zero and non-zero gradients product rules quotient rules related rates of change the relationship between the graphs of f, f', and f'' the second derivative	Find and interpret the limit at a specific value for a variety of functions (DOK3) Relate limits to derivatives in concept and for a variety of functions (DOK2) Explore functions as they change over the domain and determine the meaning for the derivative at the different points (DOK3) Investigate the relationship between a function, its derivative, and higher derivatives to better understand the nature of the function (DOK3) Differentiate between different differentiation rules for a variety of functions (DOK3)	F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
		F-BF.A.1 Write a function that describes a relationship between two quantities.
		F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
		F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes
		F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
		F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
		F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
		F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
		F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
		F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).