

**FREEHOLD REGIONAL HIGH SCHOOL DISTRICT**

**OFFICE OF CURRICULUM AND INSTRUCTION**

**MATHEMATICS DEPARTMENT**

# **GEOMETRY AND HONORS GEOMETRY**

Grade Level: 9 & 10

Credits: 5 for Geometry, 2.5 for Honors Math IA

**BOARD OF EDUCATION ADOPTION DATE:**

**August 26, 2013**

[SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING](#)

APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS

APPENDIX B: ASSESSMENT EVIDENCE

APPENDIX C: INTERDISCIPLINARY CONNECTIONS

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# Geometry & Math IA

## Course Philosophy

The Geometry and Math IA courses present the core content necessary to promote geometric proficiency and prepare students for college and careers. Students will strengthen their mathematical proficiency through problem solving, inquiry, and discovery. Students will grow to appreciate and value mathematics through the use of up-to-date resources and tools. Students will be offered the appropriate balance of analytical techniques and technological instruction as theoretical, numerical, spatial, and algebraic topics in geometry are explored.

## Course Description

These courses will formalize and extend students' geometric experiences from the middle grades. The courses are driven by the Common Core State Standards and apply the eight Mathematical Practices. Students will explore more complex geometric situations and deepen their explanations of geometric relationships moving toward formal mathematical arguments. The courses guide students to experience mathematics as a coherent, useful, and logical subject that makes use of their ability to exercise problem-solving skills in authentic situations. The curriculum includes the topics of transformations, congruence, similarity, trigonometry, two- and three-dimensional figures, circles, and probability.

Honors Math IA is an accelerated 2.5 credit semester course for the Medical Sciences Learning Center.

## Course Map

CCSS	Enduring Understandings	Essential Questions	Common Assessments
<p>G.MG.1,2,3                      G.CO.1                      N.Q.3                      G.SRT.1a,b; 2, 3, 4, 5                      G.GPE. 1,2,3(+),4,6,7                      G.SRT.6,7,8,9+,10+,11+                      G.MD.1                      G.GMD.2(+), 3, 4                      G.C.1,2,3,4(+),5</p>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p>	<ul style="list-style-type: none"> <li>• What geometric terminology can be used to describe objects?</li> <li>• How are distance and congruence related?</li> <li>• What accuracy level of a unit of measure is appropriate for given geometric calculations?</li> <li>• What theorems about triangles can be proven using properties of similar triangles?</li> <li>• How are similarity and congruence used to solve problems and prove relationships in geometric figures?</li> <li>• How are trigonometric ratios defined?</li> <li>• How do we measure a right triangle?</li> <li>• How are formulas for area, circumference of circles, and volume of solids developed and justified?</li> <li>• How is Cavalieri’s principle used to develop and justify volume formulas?</li> <li>• How are area and volume applied to density problems?</li> <li>• What geometric properties are useful in calculations of design problems such as maximum calculations?</li> <li>• What problems can be solved using volume formulas?</li> <li>• How are the cross sections of a three-dimensional figure used to generate the figure?</li> <li>• What is the two-dimensional figure that can be used to generate the three-dimensional figure through rotation?</li> <li>• What is the relationship between angles and segments of circles?</li> <li>• What can you conclude about the angles of a quadrilateral inscribed in a circle?</li> <li>• How is the relationship between tangents and circles useful in application problems?</li> <li>• What is a radian as a unit of measure?</li> <li>• How is the area of a sector derived?</li> </ul>	<p>Diagnostic Review Assessment</p> <p>Through Course Assessment #3</p> <p>Through Course Assessment #4</p> <p>Through Course Assessment #5</p> <p>Through Course Assessment #6</p>

## Course Map

CCSS	Enduring Understandings	Essential Questions	Common Assessments
<p>G.CO.2, 3, 4, 5, 6, 7, 8,9,10,11,12,13                      G.GPE. 1,2,3(+),4,5                      G.C.1,2,3,4(+),5                      G.SRT.1a,b; 2, 3, 4, 5                      G.GPE.6</p>	<p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p>	<ul style="list-style-type: none"> <li>• How do you identify transformations that are rigid motions?</li> <li>• What symmetrical patterns can be recognized in a polygon through analysis of reflection and rotation?</li> <li>• How do you draw the image of a figure under a reflection, rotation, or translation?</li> <li>• Given two congruent figures, what is the specified sequence of rigid motion that will carry one figure onto the other?</li> <li>• What does it mean for two figures to be congruent?</li> <li>• Given two triangles and the definition of congruence in terms of rigid motion, are the two triangles congruent?</li> <li>• How do the criteria for triangle congruence (ASA, AAS, SSS, SAS) follow from the definition of congruence in terms of rigid motion?</li> <li>• What properties demonstrate that all circles are similar?</li> <li>• What does it mean for two figures to be similar?</li> <li>• What criteria are essential to verify similar triangles?</li> </ul>	<p>Through Course Assessment #1                      Through Course Assessment #2                      Through Course Assessment #3                      Through Course Assessment #6</p>
<p>G.GPE. 1,2,3(+),4,5,6,7                      G.CO.8,9,10,11,12,13                      G.SRT.1a,b; 2, 3, 4, 5, 6,7,8,9+,10+,11+                      G.MD.1,2(+), 3, 4                      G.MG.2, 3                      G.C.1,2,3,4(+),5</p>	<p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	<ul style="list-style-type: none"> <li>• What is the connection between slope in regards to parallel and perpendicular lines?</li> <li>• How is a point on a directed line segment that partitions the segment in a given ratio determined?</li> <li>• What is the relationship between cosine and sine in relation to complementary angles?</li> <li>• How can you use the sine ratio to find a formula for the area of a triangle?</li> <li>• How is the Law of Sines and Cosines derived?</li> <li>• How is the Law of Sines and Cosines utilized to solve problems?</li> <li>• How are the area and perimeter of polygons on the coordinate plane calculated?</li> <li>• How is the equation of a parabola derived, given the focus and the directrix?</li> <li>• How is the equation of a circle derived given the radius and the center coordinates?</li> <li>• How is the equation of a hyperbola and an ellipse derived given the foci, using the fact that the sum or difference of distances from the foci is constant?</li> <li>• How is the coordinate plane used as a tool for geometric proofs?</li> </ul>	<p>Through Course Assessment #2                      Through Course Assessment #3                      Through Course Assessment #4                      Through Course Assessment #5                      Through Course Assessment #6</p>

## Course Map

CCSS	Enduring Understandings	Essential Questions	Common Assessments
G.GPE.5,6 G.CO.8,9,10,11,12,13 G.SRT.1a,b; 2, 3, 4, 5	Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.	<ul style="list-style-type: none"> <li>• What methods can be used to verify measures to prove theorems about lines and angles?</li> <li>• What methods can be used to verify measures to prove theorems about triangles?</li> <li>• What methods can be used to verify measures to prove theorems about parallelograms?</li> <li>• What tools and methods are used to do geometric constructions?</li> <li>• How is a regular polygon constructed?</li> <li>• How can dilations be verified experimentally?</li> </ul>	Through Course Assessment #2  Through Course Assessment #3

## Enduring Understandings & Pacing: Geometry

Unit Title	Unit Understandings	Recommended Duration
<a href="#">Review Unit</a>	An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.	0-2 weeks
<a href="#">1: Transformations</a>	The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.	4-6 weeks
<a href="#">2: Congruence</a>	<p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p> <p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p> <p>Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.</p>	4-8 weeks
<a href="#">3: Similarity</a>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p> <p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p> <p>Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.</p>	3-5 weeks
<a href="#">4: Trigonometry</a>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	5-7 weeks
<a href="#">5: Circles and Conics</a>	<p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p> <p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	5-7 weeks
<a href="#">6: Two- and Three-Dimensional Figures</a>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	5-7 weeks

## Enduring Understandings & Pacing: Honors Math IA

Unit Title	Unit Understandings	Recommended Duration
<a href="#">Review Unit</a>	An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.	0-1 week
<a href="#">1: Transformations</a>	The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.	2-3 weeks
<a href="#">2: Congruence</a>	<p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p> <p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p> <p>Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.</p>	2-4 weeks
<a href="#">3: Similarity</a>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p> <p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p> <p>Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.</p>	2-4 weeks
<a href="#">4: Trigonometry</a>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	2-3 weeks
<a href="#">5: Circles and Conics</a>	<p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p> <p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	2-3 weeks
<a href="#">6: Two- and Three-Dimensional Figures</a>	<p>An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.</p> <p>Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.</p>	2-3 weeks





**UNIT OVERVIEW**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
An understanding of the attributes of and relationships between geometric objects can be applied in diverse contexts.	What geometric terminology can be used to describe objects? How are distance and congruence related? What accuracy level of a unit of measure is appropriate for given geometric calculations?

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
Diagnostic Review Assessment	The proficient student will: <ul style="list-style-type: none"> <li>• use geometric shapes, their measures, and their properties to describe objects;</li> <li>• describe precisely the terms: angle, circle, perpendicular line, parallel line, and line segment;</li> <li>• choose a level of accuracy appropriate to the limitations on measurement when reporting quantities.</li> </ul>	G.MG.1 G.CO.1 N.Q.30

**SUGGESTED STRATEGIES**

ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
EQ1: Word wall	Perimeter Area Circumference Volume Prism Cylinder Oblique	<ul style="list-style-type: none"> <li>• Identify figures.</li> <li>• Calculate values using given formulas.</li> </ul>	Stations: students will describe geometry properties of a variety of geometric solids.
EQ2: Word wall   <b>For students who do not have declarative knowledge</b> , utilize an introduction video such as <a href="#">Geometry Chapter 1 Math Vocabulary Song</a> or <a href="#">Geometry-IHM 5th grade 2012</a> .   <b>For students who are having difficulty understanding the abstract definitions</b> , create poster/PowerPoint/video with real life examples of vocabulary words, including explanations involving the definitions.	Point Line Angle Circle Circular Arc Perpendicular & parallel line Line Segment	<ul style="list-style-type: none"> <li>• Use the terms in discussion, explanation, geometric constructions, and illustration.</li> </ul>	Vocabulary quiz with illustrations

**SUGGESTED STRATEGIES**

ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENTS
EQ3: Model measurement tools that are utilized throughout geometry; discuss answer method and types (fractions, decimals, simplified square roots, etc.) depending upon different problems	Units of measure Unit conversion Unit analysis	<ul style="list-style-type: none"> <li>Tools of measure: ruler, protractor</li> </ul>	Given measuring devices, have students identify what tool or tools can find the measurement.

**SUGGESTED MODIFICATIONS**

TECHNOLOGY INTEGRATION	
<b>Student Activities:</b> <ol style="list-style-type: none"> <li>Use Internet to define geometry vocabulary: <a href="http://www.mathwords.com">http://www.mathwords.com</a></li> <li>Use Internet to find photographs depicting the geometry vocabulary words.</li> </ol>	<b>Student Monitoring:</b> <ol style="list-style-type: none"> <li>PowerPoint presentation on geometric terms/vocabulary</li> <li>Internet quiz on vocabulary, utilizing Edmodo, QuizStar or other platform</li> </ol>
DIFFERENTIATION	
See <b>Geometry General Unit Sample Lesson</b> . <ul style="list-style-type: none"> <li>Measure angles using a protractor.</li> <li>Use the angle addition postulate in order to find the measure of an angle.</li> </ul>	

**SUPPORTING RESOURCES**

<a href="#">Appendix A: Accommodations and Modifications for Various Student Populations</a>
<a href="#">Appendix B: Assessment Evidence</a>
<a href="#">Appendix C: Interdisciplinary Connections</a>

**NJSLS Career Readiness and Preparation and Educational Technology**





<a href="#">NJSLS Career Ready Practices</a>
<a href="#">NJSLS 9.2 Career Awareness, Exploration &amp; Preparation</a>
<a href="#">NJSLS 8.1 Educational Technology</a>




**UNIT OVERVIEW**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
<p>The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.</p>	<p>How do you identify transformations that are rigid motions?                      What symmetrical patterns can be recognized in a polygon through analysis of reflection and rotation?                      How do you draw the image of a figure under a reflection, rotation, or translation?                      Given two congruent figures, what is the specified sequence of rigid motion that will carry one figure onto the other?                      What does it mean for two figures to be congruent?                      Given two triangles and the definition of congruence in terms of rigid motion, are the two triangles congruent?</p>

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
<p>Through Course Assessment #1: Transformations</p>	<p>The proficient student will:</p> <ul style="list-style-type: none"> <li>• represent transformation in a plane using transparencies and geometry software;</li> <li>• describe transformations as functions;</li> <li>• compare isometric transformations and non-isometric transformations;</li> <li>• given any quadrilateral or regular polygon, describe the rotations and reflections that carry the figure onto itself;</li> <li>• develop definitions of rotation, reflection and translation in terms of angles, circles, perpendicular lines, and parallel lines;</li> <li>• given a geometric figure and an isometric transformation, draw the transformed figure using graph paper or dynamic geometry software;</li> <li>• specify sequences that will carry a given figure onto another;</li> <li>• predict the effect of a given rigid transformation description on a figure;</li> <li>• given two figures and the definition of congruence in terms of rigid motion, decide if the two figures are congruent;</li> <li>• given two triangles and the definition of congruence in terms of rigid motion, decide if the two triangles are congruent.</li> </ul>	<p>G.CO.2                      G.CO.3                      G.CO.4                      G.CO.5                      G.CO.6                      G.CO.7</p>

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ1: Teacher will demonstrate different types of transformations with an example of each type.</p> <p> <b>For students who do not have declarative knowledge</b>, utilize introduction videos such as <a href="#">Geometry Transformations</a> and <a href="#">Khan Academy</a>.</p> <p><i>Topic to search: transformations</i></p> <p> <b>For students who have trouble visualizing abstract concepts</b>, use Geometer's Sketchpad to draw, prove and create a definition book.</p> <p> <b>For students who are having difficulty visualizing and understanding</b>, use Geometer's Sketchpad as visual aid. Other manipulatives that serve as good visuals: pinwheel (rotation), mirror (reflection), checkerboard (translations).</p> <p> <b>For teachers without Geometer's Sketchpad and who have students experiencing difficulty conceptualizing:</b></p> <p><i>Translations: Draw the same figure on two transparencies and do the transformations on the overhead as a visual.</i></p> <p><i>Reflection: Use small mirrors to reflect an object/drawing/cut-out to demonstrate a reflection. Translate the reflected image onto graph paper.</i></p> <p><i>Rotation: Cut out the shape to be rotated. Push pins/thumbtacks can act as the fixed point in center of rotation into cardboard.</i></p>	<p>Transformations Reflection Rotation Translation Dilation Describe transformation as a function – point-by-point Input- output</p>	<ul style="list-style-type: none"> <li>• Write function to represent transformation.</li> <li>• Compare and contrast rigid and non-rigid transformations.</li> </ul>	<p>Create a table of transformations patterns complete with:</p> <ol style="list-style-type: none"> <li>verbal descriptions</li> <li>function statements: ex: <math>(x,y) \rightarrow ( , )</math></li> <li>coordinate plane examples</li> </ol>

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ2: Teacher will utilize geometry software to enable students to visualize movement of figures.</p> <p> <b>For students who have perceptual impairment or difficulty visualizing,</b> label a coffee filter/paper plate with <math>0^\circ/30^\circ/45^\circ/60^\circ/90^\circ/120^\circ/135^\circ/150^\circ/180^\circ</math> increments on top of the cardboard. Cut out object and place these figures under the fixed point on top of the filter/plate. Rotate objects. Determine whether they have rotational symmetry using degree measures as a guide. Sort those that have vs. those that do not.</p>	<p>Rectangles Trapezoid Regular polygon Symmetry</p>	<ul style="list-style-type: none"> <li>Describe the rotations and reflections that carry the figure onto itself.</li> </ul>	<p>Handout with 2 quadrilaterals and 2 regular polygons: students will explain the transformations of congruent polygons that could be transformed to generate the new polygon (e.g., six transformed congruent triangles create a hexagon).</p> <p>Pattern blocks activities</p>
<p>EQ3: Large group discussion/word wall</p> <p> <b>For students who have trouble visualizing abstract concepts,</b> use Geometer's Sketchpad to draw, prove and create a definition book.</p> <p> <b>For students who are having difficulty visualizing and understanding,</b> use Geometer's Sketchpad as visual aid. Other manipulatives that serve as good visuals: pinwheel (rotation), mirror (reflection), checkerboard (translations).</p>	<p>Recall definitions: Angles Circles Perpendicular lines Line segments Reflection Rotation Translation Image Pre-image Center of rotation Line of symmetry</p>	<ul style="list-style-type: none"> <li>Describe rotation, reflection and translation in terms of angles, circles, and perpendicular lines, parallel lines.</li> </ul>	<p>Vocabulary quiz</p>
<p>EQ4: Pre-lesson: students require prior knowledge of geometry software, including basic understanding of how to create figures, label points, perform transformations, etc.</p>	<p>Isometry Transformations Rigid transformations Reflection Rotation Translation</p>	<ul style="list-style-type: none"> <li>Calculate recognized patterns of rigid transformation outcomes in the coordinate plane.</li> <li>Use dynamic geometry software and specify sequences that will carry a given figure onto another.</li> </ul>	<p>Project: given 5 geometric figures and isometric transformation directions for each, students will draw the transformed figure using graph paper and/or dynamic geometry software.</p>
<p>EQ5: Teacher-created PowerPoint/SMART Board lesson: incorporate correct terminology and notations for rigid transformations.</p>	<p>Definition of congruent figures</p>	<ul style="list-style-type: none"> <li>Use geometric descriptions of rigid motions to transform figures.</li> <li>Use geometric descriptions to predict the effect of a given motion on a figure.</li> </ul>	<p>Writing in math: have students write a viable argument and critique the reasoning of others by giving them another fictitious students' incorrect work to refute and correct.</p>

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ6: Teacher reinforces prior knowledge of geometry software, including measurement of an angle and finding the length of a side.</p> <p>Teacher reinforces prior knowledge of distance formula, slope formula, measuring using ruler and compass.</p>	<p>Identify:</p> <ol style="list-style-type: none"> <li>Corresponding pairs of sides and angles of a triangles</li> <li>Corresponding pairs of sides and angles of congruent triangles after rigid transformation</li> </ol>	<ul style="list-style-type: none"> <li>Justify congruency of two triangles after rigid transformation.</li> <li>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if corresponding pairs of sides and angles are congruent.</li> <li>Use the definition of congruence in terms of rigid motions to show that if corresponding pairs of sides and angles are congruent, then the two triangles are congruent.</li> </ul>	<p>Given pairs of triangles, students will determine if triangles are congruent or not congruent by utilizing dynamic geometry software or graph paper by showing all of the measurements are congruent/not congruent.</p>

### SUGGESTED MODIFICATIONS

#### TECHNOLOGY INTEGRATION

<p><b>Student Activities:</b></p> <ol style="list-style-type: none"> <li>Create a computer-based Jeopardy game to review each transformation.</li> <li>Utilize textbook and other videos: <a href="http://brightstorm.com/">http://brightstorm.com/</a></li> </ol>	<p><b>Student Monitoring:</b></p> <ol style="list-style-type: none"> <li>Utilize dynamic geometry software for the different transformations.</li> <li>Utilize SMART Board with Responders.</li> <li>Internet quiz on utilizing Edmodo, QuizStar or other platform</li> <li>Math worksheets: <a href="http://kutasoftware.com/">http://kutasoftware.com/</a></li> </ol>
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#### DIFFERENTIATION

<p>See <b>Geometry Unit 1 Transformations Sample Lesson Parts 1 – 3.</b></p> <ul style="list-style-type: none"> <li>Students will know that a translation is an isometry.</li> <li>Students will know that a vector can be used to represent a translation.</li> <li>Students will be able to identify translations and solve problems in the coordinate plane using translations.</li> <li>Students will use vectors to describe translations and be able to use vectors in real-life situations, such as navigation.</li> </ul>
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#### NJSLS Career Readiness and Preparation and Educational Technology

##### [NJSLS Career Ready Practices](#)

##### [NJSLS 9.2 Career Awareness, Exploration & Preparation](#)

##### [NJSLS 8.1 Educational Technology](#)

#### SUPPORTING RESOURCES

##### [Appendix A: Accommodations and Modifications for Various Student Populations](#)

##### [Appendix B: Assessment Evidence](#)




##### [Appendix C: Interdisciplinary Connections](#)

**UNIT OVERVIEW**


ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.	What is the connection between slope in regards to parallel and perpendicular lines? How do the criteria for triangle congruence (ASA, AAS, SSS, SAS) follow from the definition of congruence in terms of rigid motion? What methods can be used to verify measures to prove theorems about lines and angles? What methods can be used to verify measures to prove theorems about triangles? What methods can be used to verify measures to prove theorems about parallelograms? What tools and methods are used to do geometric constructions? How is a regular polygon constructed?
The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.	
Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.	

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
Through Course Assessment #2: Coordinate Geometry	The proficient student will: <ul style="list-style-type: none"> <li>• prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems;</li> <li>• explain how the criteria for triangle congruence (ASA, AAS, SSS, SAS) flow from the definition of congruence in terms of rigid motion;</li> <li>• prove theorems about lines and angles;</li> <li>• prove theorems about properties of triangles;</li> <li>• prove theorems about properties of sides and angle measurements of a parallelogram, including special parallelograms;</li> <li>• make a formal geometric construction using a variety of tools (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software) to copy an angle, bisect a segment, bisect an angle, construct perpendicular lines, perpendicular bisector of a segment, and a line parallel to a given line through a point not on a line;</li> <li>• construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</li> </ul>	G.GPE.5 G.CO.8 G.CO.9 G.CO.10 G.CO.11 G.CO.12 G.CO.13

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ1: Teacher will demonstrate coordinate proof and review prior knowledge of calculating slope and distance, identifying parallel and perpendicular lines, slope-intercept form and point-slope form of a line.</p> <p> <b>For students who are challenged with sequential thinking</b>, eliminate formal proof, create scaffolding, or have students cut out and order steps to complete the proof.</p>	<p>Slope Parallel lines Slope-intercept form Point-slope form Perpendicular lines</p>	<ul style="list-style-type: none"> <li>Calculate distance, slope, and identify parallel and perpendicular lines to write coordinate proofs.</li> </ul>	<p>Discover dynamic geometry software the criteria for parallel and perpendicular lines to solve geometric problems.</p>
<p>EQ2: Utilize group work to solve problems.</p> <p> <b>For students with perceptual impairment or who have difficulty visualizing</b>, demonstrate how to break apart triangles to prove congruency (two triangles connected at a vertex/ triangle within a triangle). Give students handouts to cut apart or trace onto patty paper.</p>	<p>Triangle congruence: ASA AAS SSS SAS</p>	<ul style="list-style-type: none"> <li>Identify triangle congruence given specific information.</li> </ul>	<p>Completion of problem set that asks students to:</p> <ol style="list-style-type: none"> <li>Verify congruence of given triangles given the appropriate criteria.</li> <li>Recognize required criteria missing from a verification argument.</li> </ol>
<p>EQ3: Teacher will facilitate student-created vocabulary quilt. Each student will complete “quilt block” incorporating all required vocabulary: term, student created definition, definition from geometry resource, and sketch in the four quadrants of the “quilt block”. See this <a href="#">image example of quilt</a>.</p> <p> <b>For students who are struggling with vocabulary</b>, use patty paper to facilitate student definitions. Give verbal directions using previous vocabulary to create new geometry terms. Students will create the vocabulary word through verbal directions. Students will measure to determine relationships. Example: for parallel lines, measure the distance between the lines and create a right triangle to determine slope. Have students create the definition and use the patty paper as part of their notes. Color code whenever possible.</p>	<p>Vertical angles Linear pair Parallel lines Perpendicular lines Transversals Alternate interior angles Corresponding angles Consecutive angles Interior angles Exterior angles Segment bisectors Distance formula</p>	<ul style="list-style-type: none"> <li>Calculate distance and measures of angles.</li> </ul>	<p>Student group presentations using dynamic geometry software to verify theorems</p>



SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ4: Discovery lesson</p> <p>Pre-planning: teacher will develop organizer with different types of triangles for students to calculate the sum of interior angles and midsegments.</p> <p>Students will utilize teacher-created organizer to develop and discover formulas regarding triangles, including interior angle sum of triangles, lengths of midsegments, measures of base angles of isosceles triangles, and parallel lines with respect to midsegment theorem.</p>	<p>Interior angles</p> <p>Interior angle sum</p> <p>Medians of a triangle</p> <p>Point of concurrency</p> <p>Centroid</p> <p>Isosceles triangle – base angles</p> <p>Properties of the midsegments of a triangle</p>	<ul style="list-style-type: none"> <li>Calculate:               <ol style="list-style-type: none"> <li>Sum of interior angles of a triangle or find the missing angle measure of a triangle.</li> <li>Lengths of mid-segments or length of a third side.</li> <li>Measures of angles of isosceles triangles.</li> </ol> </li> <li>Verify parallel lines in a triangle.</li> </ul>	<p>Teacher-created organizer to complete sample problems</p>
<p>EQ5: Teacher-created organizer/chart highlighting special properties of different types of quadrilaterals</p> <p> <b>Alternate to Geometer's Sketchpad for students who need tangible reinforcement:</b> <i>students measure all angles and segments to discover the properties of each quadrilateral. Check off on an organizer.</i></p>	<p>Parallelogram</p> <p>Rectangle</p> <p>Rhombus</p> <p>Square</p> <p>Opposite angles</p> <p>Diagonals</p> <p>Consecutive angles</p>	<ul style="list-style-type: none"> <li>Calculate slope and distance (length) from points on a coordinate plane.</li> <li>Verify measurements based on known properties.</li> </ul>	<p>Coordinate proof problems: verify type of quadrilateral</p>
<p>EQ6: Teacher will reinforce prior knowledge of use of ruler and protractor.</p> <p>Teacher will model introductory usage of compass.</p> <p>Teacher will create rubric for construction portfolio.</p>	<p>Geometric construction</p> <p>Straight-edge and compass</p>	<ul style="list-style-type: none"> <li>Use of geometric construction tools</li> </ul>	<p>Geometric Construction Portfolio to include:</p> <ol style="list-style-type: none"> <li>Construction examples of each: copy an angle, bisect a segment, bisect an angle, construct perpendicular lines, perpendicular bisector of a segment, and line parallel to a given line through a point not on a line</li> <li>Each students' directions or algorithm used for each construction</li> </ol>
<p>EQ7: Teacher will model usage of patty paper.</p> <p>Teacher will model inscribed regular triangle inside a circle.</p>	<p>Inscribe</p> <p>Regular polygon</p>	<ul style="list-style-type: none"> <li>Use of geometric construction tools.</li> </ul>	<p>Patty paper project</p> <p>Explore inscribing a regular hexagon in a circle.</p>

## SUGGESTED MODIFICATIONS

### TECHNOLOGY INTEGRATION

#### Student Activities:

1. Utilize dynamic geometry software to enable students to visualize movement of objects, identify parallel and perpendicular lines, congruent angle, etc.
2. Practice problems:  
<http://www.khanacademy.org/math/geometry>
3. Study guide helper:  
<http://www.cliffsnotes.com/math/geometry>

#### Student Monitoring:

1. Use SMART Responders or online software to create a practice quiz:  
<http://worksheets.theteacherscorner.net/make-your-own/math-worksheets/>, <http://quizstar.4teachers.org/>
2. Internet quiz on utilizing Edmodo, QuizStar or other platform
3. Create Geometry lessons:  
<http://www.aaamath.com/geo.htm>

### DIFFERENTIATION

See **Geometry Unit 2 Congruence Sample Lesson**.

- Prove that triangles are congruent if and only if there is a sequence of reflections, translations, and/or rotations that maps one figure onto the other.
- Use the definition of congruence to develop a method determining whether two triangles are congruent, based on comparing corresponding parts of triangles.
- Use the definition of congruence to develop shortcuts for proving that two triangles are congruent.

## SUPPORTING RESOURCES

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**UNIT OVERVIEW**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena.	How can dilations be verified experimentally? What does it mean for two figures to be similar? What criteria are essential to verify similar triangles? What theorems about triangles can be proven using properties of similar triangles? How are similarity and congruence used to solve problems and prove relationships in geometric figures? How is a point on a directed line segment that partitions the segment in a given ratio determined?
The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.	
An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts.	
Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.	

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
Through Course Assessment #3: Verifying Dilations Using Mathematical Tools	The proficient student will: <ul style="list-style-type: none"> <li>• verify by experimenting with the properties of dilations given by a center and a scale factor;</li> <li>• use the definition of similarity to decide if two figures are similar;</li> <li>• use similarity transformations to explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides;</li> <li>• use the properties of similarity transformations to establish the AA criterion for two triangles to be similar;</li> <li>• prove theorems about triangles;</li> <li>• use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures;</li> <li>• find the point on a directed line segment between two given points that partitions the segment in a given ratio.</li> </ul>	G.SRT.1.a, b G.SRT.2 G.SRT.3 G.SRT.4 G.SRT.5 G.GPE.6

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ1: Teacher will reinforce prior knowledge of geometry software.</p> <p>Teacher will lead students in creating a foldable vocabulary chart.</p>	<p>Image Pre-image Scale factor Center Dilation Reduce Enlarge Line segment Ratio</p>	<ul style="list-style-type: none"> <li>Calculate ratio and scale factors and verify experimentally.</li> </ul>	<p>Use dynamic geometry software to verify dilation properties.</p>
<p>EQ2: Teacher will model creating a word problem to illustrate real world scenarios.</p> <p> <b>For students who do not understand the concept of similarity</b>, relate to movies and familiar characters (e.g., Dr. Evil and Mini Me, or Honey I Shrank The Kids).</p> <p> <b>For students who are struggling with setting up a proportion</b>, provide labels.</p> <p>Example: <math>\frac{\text{scale}}{\text{actual}}</math> or <math>\frac{\text{little}}{\text{big}}</math></p> <p>Continually reinforce as you set up proportions. Color code whenever possible.</p> <p> <b>For students who struggle extracting information from a word problem</b>, identify the numbers and the words attached to those numbers. Underline/highlight/color code to bring relevant information to the fore. Create a drawing from the word problem.</p>	<p>Similarity Proportionality Corresponding angles Corresponding sides Similarity transformations Dilation</p>	<ul style="list-style-type: none"> <li>Calculate proportions.</li> <li>Compare and contrast angle measures and side lengths.</li> </ul>	<p>Create your own word problems to illustrate authentic scenarios showing angle and side measurements.</p>

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENTS
<p>EQ3: Discovery lesson</p> <p>Teacher will develop sequenced instructions for students to follow to discover AA criteria.</p> <p> <b>For students who need tangible examples, use patty paper/Geometer's Sketchpad to create 2 triangles based on given directions. Measure angles. Measure sides and determine if they are proportionate. Compare findings with classmates to discover AA similarity/criteria.</b></p> <p><i>Flashlight: move up and back to demonstrate dilations of reduction/enlargements.</i></p>	<p>Similar Corresponding angles Congruent angles Similarity transformation Dilation Proportionality</p>	<ul style="list-style-type: none"> <li>Recognize congruent angles.</li> <li>Calculate proportions.</li> </ul>	<p>Use dynamic geometry software to discover AA criteria.</p>
<p>EQ4: Discovery lesson</p> <p>Use dynamic geometry software or constructions to discover theorems, including: a line parallel to one side of a triangle divides the other sides proportionally and conversely; Pythagorean theorem.</p>	<p>Triangle Similarity Proportionality Pythagorean Theorem Properties of parallel lines</p>	<ul style="list-style-type: none"> <li>Simplify square roots.</li> <li>Calculate slope and proportions.</li> <li>Solve equations.</li> </ul>	<p>Use dynamic geometry software or constructions to discover theorems including:</p> <ol style="list-style-type: none"> <li>A line parallel to one side of a triangle divides the other sides proportionally and conversely.</li> <li>Pythagorean theorem, using similar right triangles</li> </ol>
<p>EQ5: Teacher will demonstrate proper techniques used in solving real-world problems.</p>	<p>Congruence Similarity</p>	<ul style="list-style-type: none"> <li>Calculate lengths of sides.</li> <li>Calculate proportions.</li> </ul>	<p>Determine the height of an object using shadows or mirrors of similar triangles.</p>
<p>EQ6: Group activity: Teacher will create different scenarios that require students to calculate ratios, distance, and slope in groups, discussing their mathematical reasoning.</p>	<p>Directed line segment Slope – rise/run Ratio Coordinate geometry Equidistant</p>	<ul style="list-style-type: none"> <li>Calculate ratios.</li> <li>Calculate distance.</li> <li>Calculate slope using rise/run.</li> </ul>	<p>Build equidistant rest stops on a highway.</p>

## SUGGESTED MODIFICATIONS

### TECHNOLOGY INTEGRATION

#### Student Activities:

1. Utilize textbook and other videos:  
<http://brightstorm.com/>
2. Practice problems:  
<http://www.khanacademy.org/math/geometry>
3. Study guide helper:  
<http://www.cliffsnotes.com/math/geometry>

#### Student Monitoring:

1. PowerPoint presentation on similarity
2. Internet quiz on utilizing Edmodo, QuizStar or other platform
3. Dynamic geometry software

### DIFFERENTIATION

See **Geometry Unit 3 Similarity Lesson Parts 1 – 2.**

- Students will identify similar polygons.
- Students will use ratio & proportions to find measures of similar polygons.
- Students will apply properties of similar polygons to solve problems.
- Students will use scale models to find the dimensions of an object being modeled.

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**UNIT OVERVIEW**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts.	How are trigonometric ratios defined? How do we measure a right triangle? What is the relationship between cosine and sine in relation to complementary angles?
Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.	How can you use the sine ratio to find a formula for the area of a triangle? How is the Law of Sines and Cosines derived? How is the Law of Sines and Cosines utilized to solve problems?

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
Through Course Assessment #4: Trigonometry	The proficient student will: <ul style="list-style-type: none"> <li>• understand that side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles;</li> <li>• explain and use the relationship between sine and cosine of complementary angles;</li> <li>• use trigonometric ratios and the Pythagorean theorem to solve right triangles in indirect measure application problems;</li> <li>• apply trigonometry to general triangles to derive the area of a triangle formula;</li> <li>• prove the Law of Sines and Cosines and use them to solve problems;</li> <li>• understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.</li> </ul>	G.SRT.6 G. SRT.7 G.SRT.8 G.SRT.9+ (Honors only) G.SRT.10+ (Honors only) G.SRT.11+ (Honors only)

SUGGESTED STRATEGIES			
ACTIVITES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ1: Teacher PowerPoint/SMART Board lesson to establish understanding of basic trigonometry ratios</p> <p>Explain utilization of trigonometric ratios to solve examples.</p> <p> <b>For students who have difficulty with retention or memorization, provide laminated copy of Trigonometric Ratio Table for reference.</b></p> <p> <b>For students who do not have declarative knowledge or need extra reinforcement, utilize videos such as <a href="#">Using SOH CAH TOA</a> and <a href="#">Pythagorean theorem</a>.</b></p>	<p>Right Triangle Similarity Acute angles Sine Cosine Tangent Side ratios</p>	<ul style="list-style-type: none"> <li>Solve right triangles given different information.</li> </ul>	<p>Create a worksheet with six authentic scenarios that use trigonometric ratios.</p>
<p>EQ2: Discovery lesson</p> <p>Construct a table demonstrating the relationship between sine and cosine of complementary angles.</p>	<p>Sine Cosine Complementary Co-function</p>	<ul style="list-style-type: none"> <li>Calculate trigonometric ratios.</li> <li>Determine the relationship between sine and cosine of complementary angles.</li> </ul>	<p>Construct a table demonstrating the relationship between sine and cosine of complementary angles.</p>
<p>EQ3: Teacher facilitates student-centered learning for students to create and use clinometers.</p> <p>Group activity</p> <p>Students will utilize clinometers to measure outdoor objects using indirect measure.</p> <p> <b>For students who need a real life application and assistance with generalizing concepts, pose the question: "How could you use trig to find the angle of depression of a sloping yard for an aboveground pool?" Give a few examples and ask students to analyze. "Could you put the pool in your yard?"</b></p>	<p>Angle of elevation Adjacent Sine Cosine Pythagorean theorem</p>	<ul style="list-style-type: none"> <li>Choose the appropriate method that could be used to solve right triangles in authentic scenarios.</li> <li>Apply right triangle trigonometric ratios and the Pythagorean theorem to solve right triangles in authentic scenarios.</li> </ul>	<p>Utilize clinometers to measure outdoor objects using indirect measure.</p>
<p>EQ4: Teacher reinforces formulas required to find the area of non-right triangles (distance, area, trigonometric ratios, etc.).</p>	<p>Trigonometric ratios Area Vertex Perpendicular</p>	<ul style="list-style-type: none"> <li>Calculate distance formula.</li> <li>Use sine, cosine, and tangent ratios for right triangles.</li> </ul>	<p>Create a poster showing no less than three examples of finding the area of a non-right triangle using the formula <math>A=1/2absin(C)</math>.</p>



SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENTS
EQ5: Teacher introduces Law of Sines and Law of Cosines and discusses when to utilize each method.	Area Law of Sines Law of Cosines	<ul style="list-style-type: none"> <li>Solve ratios.</li> <li>Calculate square roots.</li> </ul>	After researching video examples of Law of Sines and Law of Cosines, create a presentation teaching others how to use Law of Sines or Law of Cosines.
EQ6: Teacher reinforces calculating inverse trigonometric functions.	Law of Sines AAS ASA SSA Law of Cosines SSS SAS	<ul style="list-style-type: none"> <li>Identify different methods of triangle congruence.</li> <li>Calculate inverse trigonometric functions.</li> </ul>	Using the Internet, find three real-world scenarios of Law of Sines or Law of Cosines and evaluate which formula would be appropriate for use; justify your answers.

### SUGGESTED MODIFICATIONS

#### TECHNOLOGY INTEGRATION

<p><b>Student Activities:</b></p> <ol style="list-style-type: none"> <li>Students will find sine and cosine ratios of <math>30^\circ</math>, <math>45^\circ</math>, and <math>60^\circ</math> angles using the table function of a graphing calculator, then compare these values to the ones found using special right triangles.</li> <li>Use dynamic geometry software to construct similar right triangles to calculate sine, cosine, tangent in order to prove that trigonometry ratios of similar triangles are congruent.</li> </ol>	<p><b>Student Monitoring:</b></p> <ol style="list-style-type: none"> <li>Exit tickets on teacher-created PowerPoint or SMART Board presentation on Law of Sines and Cosines</li> <li>Teacher will administer SMART Responder quiz on finding basic trigonometry ratios and solving right triangles.</li> </ol>
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#### DIFFERENTIATION

<p>See <b>Geometry Unit 4 Trigonometry Sample Lesson Parts 1 – 2.</b></p> <ul style="list-style-type: none"> <li>Review their previous knowledge of the Pythagorean theorem.</li> <li>Solve a right triangle given one side and two angles of a right triangle.</li> <li>Understand that real life problems can involve trigonometry.</li> </ul>
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**UNIT OVERVIEW**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.	What properties demonstrate that all circles are similar? What is the relationship between angles and segments of circles? What can you conclude about the angles of a quadrilateral inscribed in a circle?
An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts.	How is the relationship between tangents and circles useful in application problems? What is a radian as a unit of measure? How is the area of a sector derived?
Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.	How is the equation of a parabola derived, given the focus and the directrix? How is the equation of a circle derived given the radius and the center coordinates? How is the equation of a hyperbola and an ellipse derived given the foci, using the fact that the sum or difference of distances from the foci is constant? How is the coordinate plane used as a tool for geometric proofs?

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
Through Course Assessment #6: Circles & Conics	<p>The proficient student will:</p> <ul style="list-style-type: none"> <li>• prove that all circles are similar;</li> <li>• identify and describe relationships among inscribed angles, radii, and chords;</li> <li>• identify and describe central, inscribed, and circumscribed angles;</li> <li>• recognize that the hypotenuse of an inscribed right triangle is the diameter;</li> <li>• recognize that a radius of a circle is perpendicular to the tangent where the radius intersects the circle;</li> <li>• construct the inscribed and circumscribed circles of a triangle and prove the properties of angles for a quadrilateral inscribed in a circle;</li> <li>• construct a tangent line from a point outside a given circle to the circle and recognize properties for calculations of a radius to a tangent and/or two tangents from the same external point;</li> <li>• demonstrate the ability to derive and verify using similarity, the length of an arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality;</li> <li>• demonstrate the ability to derive and verify the formula for the area of a sector as <math>s = r\theta</math> where <math>\theta</math> is the central angle in radian measure;</li> <li>• derive the equation of a parabola given a focus and directrix;</li> <li>• derive the equation of a circle given the center and radius using the Pythagorean theorem and complete the square to find the center and radius of a circle given by an equation;</li> <li>• derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from foci is constant;</li> <li>• use coordinates to prove simple geometric theorems algebraically.</li> </ul>	<p>G.C.1 G.C.2 G.C.3 G.C.4 (+) (Honors only) G.C.5 G.GPE.2 G.GPE.1 G.GPE.3 (+) (Honors only) G.GPE.4</p>

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ1: Teacher reinforces formulas pertaining to circles.</p> <p>Student-centered investigation comparing radius/diameter with circumference/area</p>	<p>Diameter, radius, area, and circumference of a circle  <math>A = \pi r^2</math>  <math>C = 2\pi r</math>            Constant of proportionality            Scale factor            Similar figures</p>	<ul style="list-style-type: none"> <li>Calculate the ratio of the circumference of a circle to the diameter for several circles (develop an understanding of <math>\pi</math>).</li> <li>Calculate constant of proportionality.</li> <li>Calculate scale factors of dilation of one circle to another using circumference.</li> </ul>	<p>Investigation of dilation of circles centered at the origin: compare circumference and areas.</p>
<p>EQ2: Word wall/word quilt/flip chart, etc.</p> <p> <b>For students who do not have declarative knowledge or are kinesthetic learners,</b> distribute cut-out circles to cooperative groups. Allow students to label circles using different materials (e.g., pipe cleaners) to represent each of the vocabulary terms.</p> <p> <b>For those who have difficulty with retention or memorization,</b> allow the use of a formula sheet.</p>	<p>Inscribed angles            Radii            Chords            Central angles            Circumscribed angles            Diameters            Tangents            Right angles            Perpendicular lines            Major arcs            Minor arcs            Intercepted arcs            Semicircles</p>	<ul style="list-style-type: none"> <li>Find the value of missing pieces of a circle given specific data. Example: given central angles find major and minor arcs.</li> </ul>	<p>Indirect measure calculation problems: cutting board problems or grill problems</p> <p>Justify reasoning of given measurements of perpendicular lines and right angles.</p>
<p>EQ3: Teacher reinforces uses of patty paper/geometry software/mathematical tools.</p> <p>Student-centered learning: students will be given different types of triangles and will be required to identify point of concurrency using different methods.</p>	<p>Inscribed in            Circumscribed about            Point of concurrency            Midpoint            Bisector            Triangle            Quadrilateral            Angle sum of a quadrilateral and triangle</p>	<ul style="list-style-type: none"> <li>Be able to use a compass, patty paper, or dynamic geometry software.</li> </ul>	<p>Use patty paper/dynamic geometry software/compass to construct the inscribed and circumscribed circles of triangles.</p>
<p>EQ4: Teacher reinforces constructions.</p>	<p>Tangent            Radius            Perpendicular bisector            Midpoint</p>	<ul style="list-style-type: none"> <li>Be able to use patty paper, compass and straight edge, or dynamic geometry software.</li> </ul>	<p>Constructions: tangent line from a point outside of the circle</p>

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENTS
EQ5: Teacher demonstrates the relationship between radians and degrees using a unit circle and circumference.	Area and circumference of a circle $A = \pi r^2$ $C = 2\pi r$ Constant of proportionality Scale factor	<ul style="list-style-type: none"> <li>Derive/explain: <math>1^\circ = \pi/180</math> radians.</li> <li>Calculate constant of proportionality using the radii (<math>r_1</math> and <math>r_2</math>) and arc lengths (<math>s_1</math> and <math>s_2</math>) of two congruent central angles <math>\frac{r_1}{s_1} = \frac{r_2}{s_2}</math>.</li> <li>Calculate radian measure.</li> <li>Calculate <math>s = r\theta</math>.</li> </ul>	<p>Write a paragraph proof and algebraically verify <math>1^\circ = \pi/180</math> radians.</p> <p>Using a large paper circle, cut each section to create congruent sections. Determine the area of each. Compare the area of the circle to the area of a parallelogram created when repasting</p>
EQ6: Teacher created lesson using PowerPoint/SMART Board/dynamic geometry software	Parabola Focus Directrix Vertex Midpoint Perpendicular Distance Formula	<ul style="list-style-type: none"> <li>Write the equation of a parabola.</li> </ul>	In teams, construct a poster of various graphs of parabolas with the focus and directrix colored to match the corresponding components in each given equation.
EQ7: Teacher aids students in derivation for formula for writing the equation of a circle not at the origin.	Distance Formula Pythagorean Theorem Complete the Square Binomial Trinomial Circle Center Radius	<ul style="list-style-type: none"> <li>Write an algebraic equation of a circle.</li> <li>Complete the square.</li> </ul>	<p>Watch the following video as a guide to derive the equation of a circle using the Pythagorean theorem:</p> <p><a href="http://www.youtube.com/watch?v=pZVufpgozCw">http://www.youtube.com/watch?v=pZVufpgozCw</a></p> <p>Create a worksheet for your peers with three sample problems.</p>
EQ8: Teacher-created lesson	Ellipse Hyperbola Foci	<ul style="list-style-type: none"> <li>Write the equation of an ellipse and a hyperbola.</li> </ul>	In teams, construct a poster of various graphs of ellipses and hyperbolas clearly showing that the sum or difference of distance from foci is constant.
EQ9: Teacher reinforces coordinate proofs, calculating distance, slope, and perpendicular lines.	Coordinate plane Vertices Distance formula Equidistant Midpoint Mid-segment of a triangle Slope Perpendicular slopes	<ul style="list-style-type: none"> <li>Write coordinate proofs.</li> <li>Calculate distance and slopes of perpendicular lines.</li> </ul>	Support geometric theorems algebraically and graphically.

## SUGGESTED MODIFICATIONS

### TECHNOLOGY INTEGRATION

#### Student Activities:

1. Students use dynamic geometry software to construct various circles and calculate ratio of circumference to diameter.
2. Students use Internet to research prices in order to spend an imaginary \$200 at a local store. They prepare a circle graph with sectors representing the amount of money spent in each predetermined category of goods (clothing, snacks, entertainment, etc.).
3. Students will watch lessons about conics as a preview to instruction: <http://www.purplemath.com/>
4. Students use graphing calculators to investigate the transformations (translations and dilations) of conic sections

#### Student Monitoring:

1. Utilize data from a SMART Responders quiz about segments of a circle as a formative assessment.
2. Collect printouts from a dynamic geometry software program to determine whether students are able to confirm theorems about chords, sectors, and tangents.

### DIFFERENTIATION

See **Geometry Unit 5 Circles and Conics Sample Lesson Parts 1 – 2.**

- Derive the equation of a circle with the given center and radius using the Pythagorean theorem and completing the square to find the center and radius of a circle given by an equation.

## SUPPORTING RESOURCES

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## NJSLS Career Readiness and Preparation and Educational Technology

[NJSLS Career Ready Practices](#)

[NJSLS 9.2 Career Awareness, Exploration & Preparation](#)


[NJSLS 8.1 Educational Technology](#)

**UNIT OVERVIEW**


ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.	How are the area and perimeter of polygons on the coordinate plane calculated? How are formulas for area, circumference of circles, and volume of solids developed and justified? How is Cavalieri’s principle used to develop and justify volume formulas? (Honors only)
An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts.	How are area and volume applied to density problems? What geometric properties are useful in calculations of design problems such as maximum calculations? What problems can be solved using volume formulas? How are the cross sections of a three-dimensional figure used to generate the figure? What is the two-dimensional figure that can be used to generate the three-dimensional figure through rotation?

**LEARNING TARGETS**

COMMON ASSESSMENT	LEARNING GOALS	CCSS
Through Course Assessment #5: 2D & 3D Figures	The proficient student will: <ul style="list-style-type: none"> <li>• use coordinates to compute perimeters of polygons and areas of triangles and rectangles;</li> <li>• make an argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone;</li> <li>• make an argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures;</li> <li>• apply concepts of density based on area and volume in modeling situations;</li> <li>• apply geometric methods to solve design problems;</li> <li>• use volume formulas to solve problems;</li> <li>• recognize and identify the shape of a two-dimensional cross section of three-dimensional objects and identify three-dimensional objects that are generated.</li> </ul>	G.GPE.7 G.G.MD.1 G.GMD.2 (+) (Honors only) G.MG.2 G.MG.3 G.GMD.3 G.GMD.4

SUGGESTED STRATEGIES			
ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENT(S)
<p>EQ1: Stations activity</p> <p>Each station will require the use of a different formula to find the area of a different figure.</p> <p> <b>For students who do not have prior knowledge of perimeter/area of irregular shapes, draw irregular shapes on graph paper. Guide students in breaking irregular shapes into regular shapes to find area.</b></p>	<p>Perimeter and area formulas of triangles and rectangles</p> <p>Distance formula</p>	<ul style="list-style-type: none"> <li>• Use coordinate geometry to find perimeter and area.</li> <li>• Calculate distance.</li> </ul>	<p>Use distance formula to find the perimeter and area of polygons in a coordinate plane.</p>
<p>EQ2: Teacher PowerPoint/SMART Board lesson covering formulas, constant of proportionality, scale factor, and similar figures</p> <p>Stations activity: each station will require students to utilize volume formulas given different figures.</p>	<p>Diameter</p> <p>Radius</p> <p>Area</p> <p>Circumference</p> <p><math>A=\pi r^2</math></p> <p><math>C=2\pi r</math></p> <p>Constant of proportionality</p> <p>Scale factor</p> <p>Similar figures</p> <p>Volume formulas: cylinder, pyramid, and cone</p>	<p>Calculate:</p> <ul style="list-style-type: none"> <li>• Ratio of the circumference of a circle to the diameter for several circles (develop an understanding of <math>\pi</math>)</li> <li>• Constant of proportionality</li> <li>• Scale factors of dilation of one circle to another using circumference</li> <li>• Volume of a cylinder, pyramid, and cone</li> </ul>	<p>Exploration: informally identify the volume of solids using popcorn, water, etc. to compare and contrast the volume of different figures.</p>
<p>EQ3: Group activity</p> <p>Teacher will facilitate penny stack method to explore Cavalieri's Principle.</p>	<p>Circumference</p> <p>Volume</p> <p>Area</p> <p>Dissection arguments Cavalieri's principle</p> <p>Informal limits arguments of: Circle, cylinder, cone, and pyramid</p>	<ul style="list-style-type: none"> <li>• Calculate circumference and area of a circle.</li> </ul>	<p>Find the volume of an oblique cylinder.</p>
<p>EQ4: Partner work requiring students to calculate surface area using nets.</p>	<p>Perimeter</p> <p>Area</p> <p>Volume</p> <p>Cylinder</p> <p>Oblique</p> <p>Sphere</p>	<ul style="list-style-type: none"> <li>• Calculate surface area and volume of spheres, cylinders and prisms.</li> </ul>	<p>Create a visual with no less than four graphics showing the differences in population density based on satellite imagery of nighttime lights.</p>

## SUGGESTED STRATEGIES

ACTIVITIES	DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	SUGGESTED ASSESSMENTS
EQ5: Teacher reinforces calculation of surface area and volume prior to student-led discovery of identifying maximum and minimum possible volume.	Volume Minimum Maximum Similarity	<ul style="list-style-type: none"> <li>Compare surface area and volume of multiple spheres, cylinders, and prisms.</li> </ul>	Describe three possible designs for a box given specific parameters. Calculate the maximum possible volume of your box designs.
EQ6: Teacher reinforces volume formulas.	Volume of: cone, cylinder, prism, pyramid, sphere	<ul style="list-style-type: none"> <li>Calculate volume of solids.</li> </ul>	Create a combination of 3 different figures consisting of a combination of cones, cylinders, prisms, pyramids, or spheres for your classmates to calculate the volume.
EQ7 & 8: Teacher models how to properly draw three-dimensional figures.   <b>For students who have perceptual impairments or are challenged by drawing 3-D figures, model drawing 3-D figures on the SMART Board or overhead. Use isometric paper and have students replicate 3-D shapes that they have created using linking cubes. Utilize Geometer's Sketchpad for rotations.</b>	Two-dimensional representation Three-dimensional representation	<ul style="list-style-type: none"> <li>Draw three-dimensional figures and identify the two-dimensional figure created by their cross sections.</li> </ul>	Sketch the three-dimensional figure generated by the rotation of a two-dimensional figure around the line of rotation.

## SUGGESTED MODIFICATIONS

### TECHNOLOGY INTEGRATION

#### Student Activities:

- Use Think3D (free app for iPads/iPhone) to enable students to construct various 3D figures:  
<http://mgleeson.edublogs.org/2012/11/28/3-ipad-apps-for-volume-and-surface-area-investigations/>
- Have students explore an interactive web site and then use graph paper to construct their own nets to share with the class:  
<http://www.learner.org/interactives/geometry/area.html>
- Students will prepare their own questions on PowerPoint for a review activity that will be presented to the class.
- Students will design a floor plan for a house with all room dimensions and research the cost of various floorings on the internet, and working within a set budget, calculate the flooring costs for their design

#### Teacher Activities:

- Internet quiz on utilizing Edmodo, QuizStar or other platform

### DIFFERENTIATION

See **Geometry Unit 6 Two and Three-dimensional Figures Sample Lesson Parts 1 – 3.**

- Use volume formulas to solve real world problems.



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