FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

MATHEMATICS DEPARTMENT

COLLEGE & CAREER MATH

Grade Level: 11-12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 29, 2016

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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COLLEGE & CAREER MATH
COURSE PHILOSOPHY
Mathematics provides us with the insight we need to understand the world around us. *College and Career Math* focuses on the application of using algebraic and geometric concepts in an advanced way to analyze and make informed decisions in authentic contexts.
COURSE DESCRIPTION
College and Career Math is a culminating course in the math sequence that builds on skills from *Algebra* and *Geometry*, strengthening students' ability to model scenarios and solve authentic problems. Problems are approached from a variety of perspectives, including graphical, numerical, verbal, and algebraic. Students will apply the standards of mathematical practices, algebraic reasoning, and statistics to analyze social applications as well as economic applications related to personal and business finance.

COURSE SUMMARY	
COURSE GOALS	
CG1: Students will solve routine and non-routine problems more efficiently by strengthening computational fluency. CG2: Students will apply and connect mathematical concepts to modern-world situations in order to make informed decisions.	
COURSE ENDURING UNDERSTANDINGS	COURSE ESSENTIAL QUESTIONS
CEU1: Computational fluency includes the ability to solve problems effectively and understand the concepts behind the computations.	CEQ1: Why is just knowing how to solve the problem not enough?
CEU2: Quantitative literacy is the ability to make sense of mathematics in authentic contexts.	CEQ2a: How can math be used to make better decisions? CEQ2b: Why is context so important to problem solving?

UNIT GOALS & PACING		
UNIT TITLE	**UNIT GOALS**	**RECOMMENDED DURATION**
Unit 1: Personal Finance	Students will analyze personal investment planning strategies to make sound financial decisions.	7-9 weeks
Unit 2: Business Mathematics	Students will apply mathematical analysis techniques to draw conclusions about successful business practices.	7-9 weeks
Unit 3: Analysis of Growth	Students will create and interpret growth models that represent authentic scenarios and make recommendations using evidence from their models.	7-9 weeks
Unit 4: Logical Analysis with Algorithms	Students will make informed decisions and evaluate the validity of claims using mathematical analysis.	7-9 weeks

COLLEGE & CAREER MATH **SUGGESTED DURATION:**
UNIT 1: Personal Finance **7-9 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will analyze personal investment planning strategies to make sound financial decisions.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can provide valuable feedback to a peer.
3	The student can: • evaluate gains and losses from stocks using percent's and dividends; • calculate interest earned in various investment vehicles; • analyze and interpret spreadsheets and graphs; • determine the average daily balance and finance charges on a credit card; • create and analyze data using scatterplots; • calculate depreciation; • analyze and calculate personal income tax; and • communicate and justify reasoning.
2	The student can complete all level 3 performances with minor mistakes and/or with assistance.
1	Even with assistance, the student cannot complete a majority of score 3 performances.
0	Even with assistance, the student cannot complete any score 3 performances.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Personal financial growth and stability is the result of sound budgeting and investments.	EQ1a: How is creating a personal budget a lifelong process? EQ1b: Can you be financially savvy and not have a budget? EQ1c: How can investments be both bad and good?
EU2: Insurance can be an essential part of sound long-term financial planning.	EQ2: Does the cost of insurance outweigh the risk of not having insurance?
EU3: Credit scores evaluate a person's creditworthiness based on previous spending and payment behaviors.	EQ3: Why do people brag about their credit score?
EU4: The value of money is dependent on many factors.	EQ4: How can monetary value be subjective?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 EU1, EQ1a, b, c EU2, EQ2 EU3, EQ3 EU4, EQ4 A.CED.A.3, A.SSE.A.1a, 1b, 2 A.SSE.B.3, 3c, FBF.A.1a, FIF.B.4, FIF.C.8b, FLE.B.5 NQ.A.1, 2, NRN.A.2, SID.B.6a SMP 1-8 DOK 4	Students will research real estate in a geographic location of their choice and choose one property to invest in. Students will then compare interest rates and mortgage points at provided lending institutions to determine the most cost efficient mortgage option. Students will write a claim in which they mathematically justify their choice in property and mortgage by using a variety of data (e.g., cost, location, time in house, taxes, insurance cost).

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
MATHEMATICS descriptive statistics domain exponential functions exponential growth and decay linear equations linear inequalities literal equations matrices measures of central tendency percent piecewise functions probability range ratios and proportions regression slope system of linear equations system of linear inequalities	Create at least two equations in two or more variables to represent and observe relationships between quantities for financial investments (DOK 2)	A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	Choose and produce an equivalent form of exponential and quadratic expressions to reveal and explain properties of the quantity represented by the original expression for financial decision making (DOK 2)	A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	Solve multi-step equations and inequalities in one variable to analyze different investment strategies (DOK 1)	A.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	Rearrange formulas to highlight a quantity of interest for a particular financial investment (DOK 1)	A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
	Calculate unit conversions to interpret data used in financial formulas (DOK 2)	NQ.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
	Determine and define appropriate quantities for the purpose of descriptive modeling of different investments (DOK 3)	NQ.A.2 Define appropriate quantities for the purpose of descriptive modeling.
	Choose and justify a level of accuracy and/or precision appropriate to limitations on measurement when reporting quantities in the context of authentic financial decision making (DOK 3)	NQ.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
FINANCE amortization CD's checking accounts compound credit cards depreciation insurance loans mortgage personal income taxes savings accounts simple interest stock market	Write a function that describes a relationship between two quantities by determining an explicit expression, a recursive process, or steps for calculation from a context of personal and financial growth (DOK 2)	FBF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
	Interpret expressions in terms of investments (DOK 2)	A.SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.
	Choose and produce an equivalent form of an exponential expression to reveal and explain properties of the quantity (DOK 2)	ASSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.
	Use the properties of exponents to interpret expressions for exponential functions in financial authentic context (DOK 2)	A.SSE.B.3c Use the properties of exponents to transform expressions for exponential functions.
	Explain how radical notation and exponents and properties of integer exponents relate to one another (DOK 2)	FIF.C.8b Use the properties of exponents to interpret expressions for exponential functions.
	Using properties of exponents, rewrite a radical expression with a rational exponent as a radical expression (DOK 1)	NRN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
	Interpret key features of finances in graphs and tables of functions in terms of the contextual quantities each function represents in investment (DOK 2)	NRN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
	Determine the viability of solutions depending on the investment model (DOK 2)	FIF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
	Rewrite expressions by identifying the structure of an expression in financial terms (DOK 1)	A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
	Recognize linear or exponential functions including vertical and horizontal shifts, vertical and horizontal dilations in financial authentic context (DOK 1)	A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.
	Recognize possible associations and trends in financial data (DOK 1)	FLE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
	Represent data with two variables on a scatter plot and describe how the variables are related (DOK 2)	SID.B.6a Use functions fitted to data to solve problems in the context of the data.
	Using matrices to represent and manipulate data in investment choices (DOK 2)	NVM.C.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

COLLEGE & CAREER MATH **SUGGESTED DURATION:**
UNIT 2: Business Mathematics **7-9 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will apply mathematical analysis techniques to draw conclusions about successful business practices.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can make recommendations for best business practices.
3	The student can: • analyze and graph profit equations; • perform a break-even analysis; • graph and interpret scatterplots; • analyze supply and demand curves; and • evaluate fixed and variable expenses;
2	The student can complete all level 3 performances with minor mistakes and/or with assistance.
1	Even with assistance, the student cannot complete a majority of score 3 performances.
0	Even with assistance, the student cannot complete any score 3 performances.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Businesses can interpret data in multiple ways based on how it is collected, organized and displayed in order to make profitable decisions.	EQ1: How can the same data lead to different conclusions? How can we be sure which conclusion will be most profitable?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 EU1, EQ1 A.CED.A.1, 2, 3, 4, A.SSE.A.1 FBF.A.1, FIF.A.1, 2, FIF.C.7b, 8, FLE.A.1, NQ.A.2, 3, REI.E.3, SID.B.6 SMP 1-8 DOK 3	Students will assume the role of an analyst for a restaurant chain. Their task will be to determine the cost of a new product. Students will analyze the supply and demand data for the product, create mathematical models of the supply and demand functions and use those models to determine the equilibrium or ideal price. They will justify their solutions mathematically.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
MATHEMATICS correlation domain & range exponential functions functions linear equations linear equations and inequalities literal expressions percent discount piecewise functions quadratic formula regression scatterplots **FINANCE** breakeven analysis dividends expense functions fixed and variable expenses revenue functions supply and demand	Create at least two equations in two or more variables to represent relationships between quantities for financial investments (DOK 2)	A.CED.A.2 Create equations that describe numbers or relationships: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	Isolate variables of quantity of interest by rewriting formulas in financial terms (DOK 1)	A.CED.A.4 Create equations that describe numbers or relationships: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
	Create and solve equations and inequalities in one variable as it relates to investment (DOK 2)	A.CED.A.1 Create equations that describe numbers or relationships: Create equations and inequalities in one variable and use them to solve problems.
	Interpret statements that use functions in terms of authentic situations, focusing on linear and exponential functions in the financial world (DOK 2)	FIF.A.2 Understand the concept of a function and use function notation: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
	Solve multi-step equations and inequalities in one variable in financial context (DOK 1)	REI.B.3 Solve equations and inequalities in one variable: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	Given a authentic situation or mathematical problem, relate the function to the context of different investments (DOK 1)	FBF.A.1 Build a function that models a relationship between two quantities: Write a function that describes a relationship between two quantities.
	Recognize situations in which one quantity changes at a constant rate per unit interval, relative to another in financial analysis (DOK 2)	FLE.A.1 Construct and compare linear, quadratic, and exponential models and solve problems: Distinguish between situations that can be modeled with linear functions and with exponential functions.
	Create at least two equations to represent relationships between quantities for financial comparison (DOK 2)	A.CED.A.2 Create equations that describe numbers or relationships: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels.
	Determine the viability of solutions according to the investment model (DOK 2)	A.CED.A.3 Create equations that describe numbers or relationships: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
	Evaluate functions for given values of x in financial situations (DOK 1)	FIF.A.1 Understand the concept of a function and use function notation: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
	Analyze the difference between simple and complex linear, quadratic, square root, ~~cube root~~, piecewise-defined, step and absolute value functions in the financial world (DOK 3)	FIF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO FURTHER DEVELOP
	Interpret different yet equivalent forms of a function, as defined by an expression in terms of a authentic financial context (DOK 2)	FIF.C.8 Analyze functions using different representations: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
	Interpret expressions in terms of the financial context (DOK 2)	A.SSE.A.1 Interpret the structure of expressions: Interpret expressions that represent a quantity in terms of its context.
	Fit a linear function for a scatter plot that suggests a linear association in the context of financial data (DOK 1)	SID.B.6 Summarize, represent, and interpret data on two categorical and quantitative variables: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
	Calculate unit conversions for financial situations (DOK 1)	NQ.A.1 Reason quantitatively and use units to solve problems: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
	Determine and define appropriate quantities for the purpose of descriptive financial modeling (DOK 2)	NQ.A.2 Reason quantitatively and use units to solve problems: Define appropriate quantities for the purpose of descriptive modeling.
	Identify important quantities in a problem or authentic financial situations (DOK 1)	NQ.A.3 Reason quantitatively and use units to solve problems: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

COLLEGE & CAREER MATH **SUGGESTED DURATION:**
UNIT 3: Analysis of Growth **7-9 WEEKS**

UNIT OVERVIEW

UNIT LEARNING GOALS

Students will create and interpret growth models that represent authentic scenarios and make recommendations using evidence from their models.

UNIT LEARNING SCALE

4	In addition to score 3 performances, the student can create authentic scenarios that can be modeled using exponential functions.
3	The student can: • create and interpret growth models that represent authentic scenarios; • graph exponential functions; • apply the exponential growth, decay, and compound interest formulas to authentic situations; • apply properties of logarithms and evaluate logarithmic expressions; and • use the properties of exponents.
2	The student can complete all level 3 performances with minor mistakes and/or with assistance.
1	With help and/or mistakes, the student can: • differentiate between exponential, linear, and quadratic graphs; • recognize the different properties of exponents; • identify variables used in the compound interest and growth/decay formulas; and • perform basic logarithmic operations.
0	Even with assistance, the student cannot complete any score 3 performances.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Growth and decay models can be used to project population and financial outcomes.	EQ1a: How accurately can math predict real events? How do you know when to trust them?
EU2: The analysis of quantities and their relationships are used to make informed decisions.	EQ2: How can math be used to make better decisions?

COMMON ASSESSMENT

ALIGNMENT	DESCRIPTION
LG 1 EU1, EQ1A EU2, EQ2 A.SSE.1, 3.c, A.REI.11, F.BF.1.b, F.IF.4-7, F.LE.3, S.ID.7 SMP 1-8 DOK 3	You just recently got married and received $30,000 cash in wedding gifts. Research several financial institutions to help make an informed decision on which bank and type of account will yield the most interest. State a claim regarding which bank you choose and why. Final analysis should include financial institutions researched, type of account, annual interest rate, equations that represent total amounts after 10, 20 and 30 years and the percent increase for each institution.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
PROPERTIES OF EXPONENTS exponent quotient of a powers property negative power of a power property power of a product property zero exponent power of a quotient property product of powers property properties of exponents **GROWTH AND DECAY** exponential decay exponential growth formula for compound interest formulas for exponential growth and decay growth and decay factors mathematical constant *e* parent graph for exponential growth and decay models percent **LOGARITHMIC FUNCTIONS** change of base formula properties of logarithmic functions	Use the properties of exponents to interpret expressions for exponential functions (DOK 1) Use the properties of exponents to rewrite a radical expression using a rational exponent and vice versa (DOK 1)	A.SSE.3.c Use the properties of exponents to transform expression for exponential functions.
	Solve linear and exponential equations in one variable (DOK 1)	A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	Graph exponential, logarithmic, and trigonometric functions, by hand and using technology (DOK 2) For exponential and logarithmic functions show intercepts and end behavior (DOK 1)	FIF.7e Graph exponential and logarithmic functions showing intercepts and end behavior.
	Prove that linear functions grow by equal differences over equal intervals (DOK 3)	FLE.1.a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
	Prove that exponential functions grow by equal factors over equal intervals (DOK 3)	F.LE.1.c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
	Identify information needed for exponential growth and decay problems (DOK 2)	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.
	Evaluate and solve logarithmic and exponential equations. Apply growth and decay formulas to authentic models (DOK 2)	A.CED.1 Create equations and inequalities in one variable and use them to solve problem to Include equations arising from exponential functions.
	Create equations (linear and exponential) and inequalities in one variable and use them to solve problems (DOK 2) Solve exponential equations using logarithms (DOK 2) Write a quadratic function and explain various properties of the function to determine which form of the quadratic is the most appropriate (DOK 2) Determine which form of the function is the most appropriate for interpretation in a authentic context (DOK 2)	FLE.2 Construct and compare linear, quadratic, and exponential models and solve problems.
	Sketch graphs showing the key features of a function and modeling relationships between two quantities, given a verbal description of the relationship (DOK 1)	FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

COLLEGE & CAREER MATH **SUGGESTED DURATION:**
UNIT 4: Logical Analysis With Algorithms **7 – 9 WEEKS**

UNIT OVERVIEW	
UNIT LEARNING GOALS	
Students will make informed decisions and evaluate the validity of claims using mathematical analysis.	
UNIT LEARNING SCALE	
4	In addition to score 3 performances, the student can provide valuable feedback to a peer.
3	The student can: • use cryptology to encode and decode message; • find an object in nature and explain how the golden ratio is represented by the object; • use data to represent different views of a given topic; and • use trees for an analysis of e-mail travel.
2	The student can complete all level 3 performances with minor mistakes and/or with assistance.
1	Even with assistance, the student cannot complete a majority of score 3 performances.
0	Even with assistance, the student cannot complete any score 3 performances.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
EU1: Mathematical analysis is an integral part of interpreting data and the validity of claims made.	EQ1: How can math be used to manipulate or strengthen an argument?
EU2: Quantitative analysis can be used to make efficient choices.	EQ2: How can math make our lives more efficient?
EU3: Algorithms are an important process in quantitative analysis and data organization.	EQ3: How necessary are algorithms? Can you be effective without them?

COMMON ASSESSMENT	
ALIGNMENT	DESCRIPTION
LG 1 EU1, EQ1 EU2, EQ2 EU3, EQ3 G.SRT.1, 5, S.IC.6, N.VM.6, 7, 8, 9 SMP 1-8 DOK3	Students will complete the three independent assessments: 1. Students will use the golden ration to analyze the face of a famous person. Students will develop an argument based on the attractiveness of the person using their mathematical analysis. 2. Students will develop an argument about the accuracy of utilizing small scale polls to predict larger voting trends. 3. Students will use cryptology to read an encoded message.

TARGETED STANDARDS		
DECLARATIVE KNOWLEDGE	PROCEDURAL KNOWLEDGE	STANDARDS TO INTRODUCE
binary tree equivalent expressions factoring golden ratio margin of error matrix operations paths proportion ratio sample size similar triangles similarity statistical significance survey symmetry trees	Utilize ratio and proportion to derive the golden ratio (DOK 3) Determine building heights using shadows and similar triangles (DOK 2)	G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
	Model the golden ratio in nature, architecture, and advertising (DOK 3)	G.SRT.1 Students will verify experimentally the properties of dilations given by a center and a scale factor.
	Use tree diagrams for practical applications including analysis of email travel, three dimensional computer graphics, and spell checkers (DOK 3)	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
		N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.
	Compare and contrast methods of data interpretation to determine validity (DOK 2) Students will calculate experimental and theoretical statistical data (DOK 3)	S.IC.6 Make inferences and justify conclusions form sample surveys, experiments and observational studies. Evaluate reports based on data.
	Use cryptology to create encoded messages (DOK 3)	N.VM.6 Use matrices to represent and manipulate data, eg., to represent payoffs or incidents relationships in a network.
		N.VM.7 Multiply matrices by scalars to produce new matrices eg., as when all of the payoffs in a game are doubled.
		N.VM.8 Add, subtract and multiply matrices of appropriate dimensions.
		N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square numbers is not a commutative operation, but still satisfies the associative and distributive properties.
	Create box plot, histogram and other data representations of research of authentic data (DOK4)	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).