

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

MATHEMATICS DEPARTMENT

CALCULUS

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 22, 2011

[SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING](#)

APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS

APPENDIX B: ASSESSMENT EVIDENCE

APPENDIX C: INTERDISCIPLINARY CONNECTIONS

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Calculus - Introduction

Introduction

Course Philosophy

Students will learn to appreciate the value of calculus as the mathematics of change and motion. This course will prepare students for further study in all branches of higher mathematics, science, and related fields. Calculus is not only the language for expressing physical laws in precise mathematical terms, but it is also a tool for studying these laws. The course emphasizes an approach to calculus that involves problems being expressed numerically, analytically, graphically, and verbally. Technology is used regularly to reinforce these approaches, to confirm written work, to implement experimentation, and to assist in interpreting results.

Course Description

This course is intended for students interested in business, economics, technology, and other related fields. Topics covered include functions and graphs, tangent lines, derivatives, limits and continuity, and applications of differentiation and integration. Calculus provides the answer to questions that cannot be solved by using, algebra, geometry, or trigonometry alone.

Course Map and Proficiencies/Pacing

Course Map

Relevant Standards	Enduring Understandings	Essential Questions	Assessments		
			Diagnostic	Formative	Summative
MA.9-12.F-IF.4 MA.9-12.F-IF.5 MA.9-12.F-IF.7 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.1	Analysis of the critical elements of functions is essential to calculus.	How are functions used in problem solving when modeling real world situations? How are trigonometric functions used to model real world situations?	Pre-test	Do Now assignments Questions & answers Closure questions	Unit test (multiple choice & free response questions) Alternative assessment
MA.9-12.F-IF.1 MA.9-12.F-IF.2 MA.9-12.F-IF.4 MA.9-12.F-IF.7	Functions can be analyzed graphically by their limiting behavior and rates of change.	How does the concept of a limit lead to a derivative? How are derivatives used to analyze the behavior of a function? How does local linearity allow us to evaluate a limit?	Pre-test	Do Now assignments Questions & answers Closure questions	Unit test (multiple choice & free response questions) Alternative assessment
MA.9-12.F-IF.2 MA.9-12.F-IF.3 MA.9-12.F-BF.1 MA.9-12.F-BF.2 MA.9-12.F-LE.5	The Fundamental Theorem of Calculus connects differential and integral calculus.	How do differential equations help to describe rates of change? How are integrals used to measure changing quantities?	Pre-test	Do Now assignments Questions & answers Closure questions	Unit test (multiple choice & free response questions) Alternative assessment

MA.9-12.F-IF.2 MA.9-12.F-IF.4 MA.9-12.F-IF.5 MA.9-12.F-BF.1 MA.9-12.F-BF.3	Limits are the underlying concept supporting physical applications that are imbedded in many fields.	How is calculus used to solve optimization and related rate problems? How are the area of bounded regions and volume of solids evaluated?	Pre-test	Do Now assignments Questions & answers Closure questions	Unit test (multiple choice & free response questions) Alternative assessment
MA.9-12.F-IF.1-9 MA.9-12.F-BF.1-5 MA.9-12.F-IF.7 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.1	Math can be communicated verbally, graphically, and using technology.	In what ways can the analytical work of calculus be supported? How is a graphing calculator used as a problem solving tool?	Pre-test	Do Now assignments Questions & answers Closure questions	Unit test (multiple choice & free response questions) Alternative assessment

Proficiencies and Pacing

Unit Title	Unit Understanding(s) and Goal(s)	Recommended Duration
Unit 1: Functional Analysis	Analysis of the critical elements of functions is essential to calculus. At the conclusion of this unit, students will be able to: 1. Compare and contrast functions. 2. Use functions to model real life situations.	4 weeks
Unit 2: Limits and Continuity	Functions can be analyzed graphically by their limiting behavior and rates of change. At the conclusion of this unit, students will be able to: 1. Evaluate limits. 2. Determine graphical behavior and continuity.	4 weeks
Unit 3: Derivatives	Functions can be analyzed graphically by their limiting behavior and rates of change. At the conclusion of this unit, students will be able to: 1. Evaluate Derivatives. 2. Find the equation of a tangent line.	6 weeks

Unit 4: Applications of Derivatives	<p>Limits are the underlying concept supporting physical applications that are imbedded in many fields.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Solve optimization and related rate problems. 2. Analyze the behavior of a function. 	5 weeks
Unit 5: The Definite Integral	<p>The Fundamental Theorem of Calculus connects differential and integral calculus.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Evaluate definite integrals. 2. Find the area under a curve. 	5 weeks
Unit 6: Differential Equations	<p>The Fundamental Theorem of Calculus connects differential and integral calculus.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Solve differential equations. 	5 weeks
Unit 7: Applications of Integrals	<p>Limits are the underlying concept supporting physical applications that are imbedded in many fields.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Find the area and volume of a curve. 	5 weeks

Calculus - Unit 01

Unit 1: Functional Analysis

Enduring Understandings:

Analysis of the critical elements of functions is essential to calculus.

Essential Questions:

How are functions used in problem solving when modeling real world situations?

How are trigonometric functions used to model real world situations?

Unit Goals:

1. Compare and contrast functions.
2. Use functions to model real life situations.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can you compare the graphs of piecewise, absolute value, and composite functions?	Define function, domain, range, even function, odd function, and symmetry Graph examples of piecewise and absolute value functions Find the value of composite functions	Diagnostic test		Written tests quizzes
		Worksheets and samples	Lecture and class discussion	Worksheets
		Teacher driven worksheets	Guided and independent practice	Project assessments
		Textbook ancillaries	Worksheets to reinforce the concepts	Responses to discussion questions
		Multimedia presentation	Guided and independent practice using TI-83	Journal assessments
		Graphing calculators		Closure questions
		Dry erase boards		Exit slips
Geometer's Sketchpad				

<p>What are the differences between the models and graphs of exponential growth and exponential decay?</p>	<p>Represent the exponential function with base a</p> <p>Compare exponential growth and decay</p> <p>Represent the exponential function with the natural base e</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentation</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Lecture and class discussion</p> <p>Guided and independent practice</p> <p>Worksheets to reinforce the concepts</p> <p>Guided and independent practice using TI-83</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>How does one identify and find an inverse of a function?</p>	<p>Define a one-to-one function</p> <p>Test for inverses graphically</p> <p>Write the inverse as a function of x</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentation</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Lecture and class discussion</p> <p>Guided and independent practice</p> <p>Worksheets to reinforce the concepts</p> <p>Use white boards to show immediate feedback on questions</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

<p>How are the properties of logarithms used to solve equations?</p>	<p>Use the properties of logarithms to solve equations</p> <p>Solve equations involving logarithmic and exponential expressions</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentation</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Guided and independent practice</p> <p>Worksheets to reinforce the concepts</p> <p>Use white boards to show immediate feedback on questions</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>What is the relationship between the six trigonometric functions, their graphs, and the unit circle?</p>	<p>Define and convert radian measure</p> <p>Graph the six trigonometric functions and observe the domain, range, and period</p> <p>Graph a trigonometric function using transformations</p> <p>Use trigonometric identities to verify equations</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentation</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Lecture and class discussion</p> <p>Guided and independent practice</p> <p>Worksheets to reinforce the concepts</p> <p>Complete the chapter study guides</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

MA.9-12.F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MA.9-12.F-IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MA.9-12.F-IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

MA.9-12.F-BF.1

Write a function that describes a relationship between two quantities.

Differentiation

Jig saw lesson: Student groups can create presentations on how to graph each one of the six trigonometric functions' parent graphs and their inverse, and present the group's work to the class.

Technology

The graphing calculators are useful tools when recognizing the patterns in graphs of functions.

College and Workplace Readiness

Medical professionals use sinusoidal graphs to determine the health of an individual on monitors.

Calculus - Unit 02

Unit 2: Limits

Enduring Understandings:

Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:

How does the concept of a limit lead to a derivative?

How are derivatives used to analyze the behavior of a function?

Unit Goals:

1. Evaluate limits.
2. Determine graphical behavior and continuity.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
What are the graphical, numerical, and analytical ways to determine a limit of a function at a given value and its continuity at that point?	Define limit, one sided limits, properties of limits	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad	Zeno's Paradox	Written tests quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

<p>How can a limit be used to determine the graphical behavior of a function?</p>	<p>Identify continuity, end-behavior models, hole(s), asymptotes, oscillating functions, one-sided limits, and jump discontinuity in functions</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Graphing calculator exploration of graphs</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>How can the limit of the average rate of change of a function lead to the instantaneous rate of change?</p>	<p>Apply average rate of change, secant line, instantaneous rate of change, tangent line, slope at a point, normal line to real world problems</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Teacher created worksheet</p> <p>Interactive white board presentation</p> <p>Class discussion</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

MA.9-12.F-IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Students can create games (bingo, flashcards, monopoly, etc.) to demonstrate their understanding of classifying functions as continuous or discontinuous.

Technology

Teacher will use graphing software programs such as WinPlot and the graphing calculators to reinforce the concept of limits and how it leads to the slope of a tangent line.

College and Workplace Readiness

When working with compound interest and profit functions, a banker needs to understand the limiting behaviors of business models.

Calculus - Unit 03

Unit 3: Derivatives

Enduring Understandings:

Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:

How are derivatives used to analyze the behavior of functions?

Unit Goals:

1. Evaluate Derivatives.
2. Find the equation of a tangent line.

Recommended Duration: 6 weeks

Guiding/Topical Questions	Content/Themes/kills	Resources and Materials	Suggested Strategies	Suggested Assessments
How does the definition of a derivative lead to the analysis of graphs?	<p>Apply the definition of a derivative. Discuss the different notations of a derivative</p> <p>Graph a function and its derivative</p> <p>Find a right-hand and left-hand derivative of a function</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	Lecture and class discussion	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

<p>What are the conditions for differentiability?</p>	<p>Illustrate the four different instances where a derivative fails to exist</p> <p>Use the Intermediate Value Theorem and the Mean Value Theorem to show differentiability implies continuity</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Lecture and class discussion</p> <p>Guided and independent practice</p> <p>Worksheets to reinforce the concepts</p> <p>Use white boards to show immediate feedback on questions</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>What are the various rules for differentiation and how to apply them?</p>	<p>Apply rules for differentiation such as Power rule (positive and negative integer powers), Constant Multiple rule, Sum and Difference rule, Product rule, Quotient rule, and Chain rule</p> <p>Compute derivatives of transcendental functions</p> <p>Calculate and apply second and higher order derivatives</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Lecture and class discussion</p> <p>Guided and independent practice</p> <p>Worksheets to reinforce the concepts</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>How does the derivative relate to velocity and acceleration?</p>	<p>Define acceleration and velocity in terms of a derivative</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Lecture and class discussion</p> <p>Odometer project</p>	<p>Written tests quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

How is implicit differentiation an important tool in finding derivatives?	Differentiate a multivariable function using implicit differentiation Define the process of implicit differentiation	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad	Lecture and class discussion Complete the chapter study guides	Written tests quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips
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MA.9-12.F-IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Students can create flashcards of the derivative graphs of the function graphs provided by the teacher. Students can then get into groups where they can match function graphs to their derivative graphs.

Technology

Calculus in Motion® in conjunction with Geometer's Sketchpad® can be used to understand what the graph of a derivative will look like.

College and Workplace Readiness

Biologists use the instantaneous rate of change at a point to analyze population growth rate.

Calculus - Unit 04

Unit 4: Applications of Derivatives

Enduring Understandings:

Limits are the underlying concept supporting physical applications that are imbedded in many fields.

Essential Questions:

How is calculus used to solve optimization and related rate problems?

Unit Goals:

1. Solve optimization and related rate problems.
2. Analyze the behavior of a function.

Recommended Duration: 5 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How does one determine a function's maximum or minimum value?	Determine critical points such as extreme values of a function (relative extrema, and absolute extrema), and points of inflection	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad	Lecture and class discussions Guided and independent practice using the TI-83	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips Cooperative learning sessions Alternative assessment Teacher observation

<p>How does one determine relative extrema and intervals on which a function is increasing or decreasing?</p>	<p>Identify increasing function and decreasing function intervals Apply first derivative test and Mean Value Theorem</p>	<p>Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad</p>	<p>Matching graphs of derivatives and functions</p>	<p>Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips Cooperative learning sessions Alternative assessment Teacher observation</p>
<p>How does one determine concavity?</p>	<p>Determine points of inflection Use the second derivative to determine concavity</p>	<p>Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad</p>	<p>Lecture and class Discussions Use whiteboard to check for understanding of concept</p>	<p>Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips Cooperative learning sessions Alternative assessment Teacher observation</p>

<p>How are the rates of changing quantities related?</p>	<p>Use applications involving changing quantities and related rates</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Class discussion</p> <p>Discovery based textbook activity</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> <p>Cooperative learning sessions</p> <p>Alternative assessment</p> <p>Teacher observation</p>
<p>How is the derivative used to optimize quantities?</p>	<p>Use applications involving maximizing/minimizing quantities for optimization</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Build a box of maximum volume</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> <p>Cooperative learning sessions</p> <p>Alternative assessment</p> <p>Teacher observation</p>

MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.
MA.9-12.F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Differentiation

Process differentiation: Students will self test using one of the practice tests available (internet based or pencil & paper).

Technology

The graphing calculator can be used to understand the behavior of functions.

College and Workplace Readiness

Agriculturalists use optimization to maximize/minimize dimensions of a field to be more efficient with their land usage.

Calculus - Unit 05

Unit 5: The Definite Integral

Enduring Understandings:

The Fundamental Theorem of Calculus connects differential and integral calculus.

Essential Questions:

How are integrals used to measure changing quantities?

Unit Goals:

1. Evaluate definite integrals.
2. Find the area under a curve.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can the definite integral be applied to finding area under a curve?	Use approximation methods (LRAM, RRAM, MRAM, and Trapezoidal) to find area under a curve	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad	Bank Robbery project	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

<p>How does estimating finite sums lay the foundation for finding the value of a definite integral?</p>	<p>Apply Summation formulas to evaluate a finite/infinite Riemann Sums</p> <p>Apply properties of a definite integral</p> <p>Apply integral computations of area problems</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Map project</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>How does the limit of a Riemann Sum relate to integration?</p>	<p>Define integral, Integrand, Integral function</p> <p>Apply the properties of a definite integral</p> <p>Evaluate a definite integral using area</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Odometer project part 2</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

<p>How does the Fundamental Theorem of Calculus connect integral and differential calculus?</p>	<p>Understand how the Fundamental Theorem of Calculus, anti-derivatives, indefinite integrals, and Mean Value (Average) Theorem for integration and connect integral and differential Calculus</p>	<p>Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad</p>	<p>Interactive white board presentation/class discussion</p>	<p>Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips</p>
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- MA.9-12.F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- MA.9-12.F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- MA.9-12.F-BF.1 Write a function that describes a relationship between two quantities.
- MA.9-12.F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- MA.9-12.F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Differentiation

Content Differentiation: Discovery learning- The students are given a graph with a pre-drawn curve on the graph paper and are asked to develop a strategy to calculate the area between the curve and the x - axis using geometry.

Technology

Geometer's Sketchpad® is an excellent tool for student and/or teacher demonstrations of the essential components of integration.

College and Workplace Readiness

Scientists and actuaries use integration to analyze and calculate statistical data.

Calculus - Unit 06

Unit 6: Differential Equations

Enduring Understandings:

The Fundamental Theorem of Calculus connects differential and integral calculus.

Essential Questions:

How are differential equations used to describe rates of change?

Unit Goals:

1. Solve differential equations.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How changing quantities are described using differential equations?	Graph and analyze slope fields and compare relationships to differentiable equations and indefinite integrals	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad	Murder mystery problem Newton's Law of Cooling Spread of a disease	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

<p>What is the relationship between slope fields and differential equations?</p>	<p>Identify the relationship between slope fields and differential equations</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Supplemental AP materials Interactive white board presentations/ class discussion</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>What are some alternative techniques of integration?</p>	<p>Understand and utilize alternative techniques of integration such as Partial Fractions, Integration by Parts, and Trigonometric Substitution</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Cooperative learning using teacher created worksheets</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.2
MA.9-12.F-IF.3
MA.9-12.F-BF.1
MA.9-12.F-BF.2
MA.9-12.F-LE.5

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
Write a function that describes a relationship between two quantities.
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Interpret the parameters in a linear or exponential function in terms of a context.

Differentiation

Process Differentiation: Students can investigate the various techniques used for integration on internet tutorials sites and submit findings of their investigations for review.

Technology

The World Wide Web offers a variety of perspectives on techniques and applications of integration for students' enrichment.

College and Workplace Readiness

Meteorologists utilize slope fields in weather maps.

Calculus - Unit 07

Unit 7: Applications of Integrals

Enduring Understandings:

Limits are the underlying concept supporting physical applications that are imbedded in many fields.

Essential Questions:

How are the area of bounded regions and volume of solids evaluated?

Unit Goals:

1. Find the area and volume of a curve.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can one estimate and calculate the area of the bounded region between two curves?	Estimate area under a curve using the area of rectangle and relate to definite integrals	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing calculators Dry erase boards Geometer's Sketchpad	Interactive white board presentation/class discussion	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

<p>How can the definite integral be used to calculate the volume of a solid with a known cross section?</p>	<p>Use Area formulas to calculate the volume of a solid with a known cross section and relate it to definite integrals</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Rice Krispies treats project</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>
<p>How can the definite integral be used to calculate the volume of a solid obtained by rotating a plane region?</p>	<p>Use Disc/Washer Method and Shell Method to calculate the volume of a solid obtained by rotation a plane region and relate it to definite integrals</p>	<p>Diagnostic test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p>	<p>Model volume through candy kisses, cakes, ice cream cones</p>	<p>Written tests and quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p>

MA.9-12.F-IF.2
MA.9-12.F-IF.4

MA.9-12.F-IF.5
MA.9-12.F-BF.1
MA.9-12.F-BF.3

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Write a function that describes a relationship between two quantities.
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Differentiation

Product Differentiation: Students can create a solid and calculate the volume using each of the methods described above or research an application of the volume model and create a poster demonstrating appropriate methods for volume calculation.

Technology

The graphing calculators are utilized to represent or demonstrate an understanding of integration. Mathematica® software brings the three dimensional model to life.

College and Workplace Readiness

Using Calculus to determine areas and volumes of regions bounded by curves is utilized by mechanical engineers for dam and water tower designs and construction.