FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

MATHEMATICS DEPARTMENT

AP CALCULUS BC

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 22, 2011

SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING
APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS
APPENDIX B: ASSESSMENT EVIDENCE
APPENDIX C: INTERDISCIPLINARY CONNECTIONS

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AP Calculus BC - Introduction

Introduction

Course Philosophy

Students will learn to appreciate the value of calculus as the mathematics of change and motion. This course will prepare students for further study in all branches of higher mathematics, science, and related fields. Calculus is not only the language for expressing physical laws in precise mathematical terms, but it is also a tool for studying these laws. The course emphasizes an approach to calculus that involves problems being expressed numerically, analytically, graphically, and verbally. Technology is used regularly to reinforce these approaches, to confirm written work, to implement experimentation, and assist in interpreting results.

Course Description

Calculus BC is primarily concerned with developing the students' understanding of the concepts of introductory and advanced calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are also important.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Although manipulation and computational competence is an important outcome, it is not the core of this course.

Technology will be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, applications, and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics.

Course Map and Proficiencies/Pacing

Course Map

Relevant Standards	Enduring Understandings	Essential Questions	Assessments		
			Diagnostic	**Formative**	**Summative**
MA.9-12.F-IF.4 MA.9-12.F-IF.5 MA.9-12.F-IF.7 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.1	Analysis of the critical elements of functions is essential to calculus.	How are functions used in problem solving when modeling real world situations? How are trigonometric functions used to model real world situations?	Remote responder Quiz	Do Now assignments Questions & answers Closure questions Group tests Remote responder Quiz	Unit test (multiple choice & free response questions) Alternative assessment Take home tests
MA.9-12.F-IF.1 MA.9-12.F-IF.2 MA.9-12.F-IF.4 MA.9-12.F-IF.7	Functions can be analyzed graphically by their limiting behavior and rates of change.	How does the concept of a limit lead to a derivative? How are derivatives used to analyze the behavior of a function? How does local linearity allow us to evaluate a limit?	Remote responder Quiz	Do Now assignments Questions & answers Closure questions Group tests Remote responder Quiz	Unit test (multiple choice & free response questions) Alternative assessment Take home tests
MA.9-12.F-IF.2 MA.9-12.F-IF.3 MA.9-12.F-BF.1 MA.9-12.F-BF.2 MA.9-12.F-LE.5	The Fundamental Theorem of Calculus connects differential and integral calculus.	How do differential equations help to describe rates of change? How are integrals used to measure changing quantities.	Remote responder Quiz	Do Now assignments Questions & answers Closure questions Group tests Remote responder Quiz	Unit test (multiple choice & free response questions) Alternative assessment Take home tests

MA.9-12.F-IF.2 MA.9-12.F-IF.4 MA.9-12.F-IF.5 MA.9-12.F-BF.1 MA.9-12.F-BF.3	Limits are the underlying concept supporting physical applications that are imbedded in many fields.	How is calculus used to solve optimization and related rate problems? How are the area of bounded and unbounded regions and volume of solids evaluated? How motion is analyzed using parametric equations and vectors? How are slope and area measured in a polar coordinate system? How are transcendental functions described using infinite series?	Remote responder quiz	Do Now assignments Questions & answers Closure questions Group tests Remote responder Quiz	Unit test (multiple choice & free response questions) Alternative assessment Take home tests
MA.9-12.F-IF.3 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.2	Power series are important tools for approximating and defining functions.	How is the convergence of an infinite series determined? How does a Taylor Series represent a function?	Remote responder quiz	Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz	Unit test (multiple choice & free response questions) Alternative assessment Take home tests
MA.9-12.F-IF.1-9 MA.9-12.F-BF.1-5 MA.9-12.F-IF.7 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.1	Math can be communicated verbally, graphically, and using technology.	In what ways can the analytical work of calculus be supported? How is a graphing calculator used as a problem solving tool?	Remote responder quiz	Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz	Unit test (multiple choice & free response questions) Alternative assessment Take home tests

Proficiencies and Pacing

Unit Title	Unit Understanding(s) and Goal(s)	Recommended Duration
Unit 1: Functional Analysis	Analysis of the critical elements of functions is essential to calculus. At the conclusion of this unit, students will be able to: 1. Compare and contrast functions. 2. Use functions to model real life situations.	.5 week
Unit 2: Limits and Continuity	Functions can be analyzed graphically by their limiting behavior and rates of change. At the conclusion of this unit, students will be able to: 1. Evaluate limits. 2. Determine graphical behavior and continuity.	1.5 weeks
Unit 3: Derivatives	Functions can be analyzed graphically by their limiting behavior and rates of change. At the conclusion of this unit, students will be able to: 1. Evaluate derivatives. 2. Find the equation of a tangent line.	2 weeks
Unit 4: Applications of Derivatives	Limits are the underlying concept supporting physical applications that are embedded in many fields. At the conclusion of this unit, students will be able to: 1. Solve optimization and related rate problems. 2. Analyze the behavior of a function.	4 weeks
Unit 5: Definite Integrals	The Fundamental Theorem of Calculus connects differential and integral calculus. At the conclusion of this unit, students will be able to: 1. Evaluate definite integrals. 2. Find the area under a curve.	2 weeks

Unit 6: Differential Equations	The Fundamental Theorem of Calculus connects differential and integral calculus. At the conclusion of this unit, students will be able to: 1. Solve differential equations.	2 weeks
Unit 7: Applications of the Definite Integral and Mathematic Modeling	Limits are the underlying concept supporting physical applications that are embedded in many fields. At the conclusion of this unit, students will be able to: 1. Find the area and volume of a curve.	3 weeks
Unit 8: Improper Integrals	Limits are the underlying concept supporting physical applications that are embedded in many fields. At the conclusion of this unit, students will be able to: 1. Evaluate limits for indeterminate form. 2. Determine the convergence of improper integrals.	2 weeks
Unit 9: Infinite Series	Power series are important tools for approximating and defining functions. At the conclusion of this unit, students will be able to: 1. Represent functions using Taylor Series. 2. Determine the interval of convergence. 3. Estimate the error bound when evaluating a function with a Taylor Polynomial.	4 weeks
Unit 10: Parametric/Polar Equations	Limits are the underlying concept supporting physical applications that are embedded in many fields. At the conclusion of this unit, students will be able to: 1. Use parametric equations to analyze motion with velocity and acceleration vectors. 2. Find areas of regions defined by polar equations.	2 weeks
Unit 11: AP Review	Math can be communicated verbally, graphically, and using technology. At the conclusion of this unit, students will be able to: 1. Communicate mathematics in written form.	4 weeks

AP Calculus BC - Unit 01

Unit 1: Functional Analysis

Enduring Understandings:

Analysis of the critical elements of functions is essential to calculus.

Essential Questions:

How are functions used in problem solving when modeling real world situations?

How are trigonometric functions used to model real world situations?

Unit Goals:

1. Compare and contrast functions.
2. Use functions to model real life situations.

Recommended Duration: .5 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can you compare the graphs of piecewise, absolute value, and composite functions?	Define: function, domain, range, even function, odd function, and symmetry Graph examples of piecewise and absolute value functions Find the value of composite functions	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Guided and independent practice using the TI 83	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

What are the differences between the models and graphs of exponential growth and exponential decay?	Represent the exponential function with base a Compare exponential growth and decay Represent the exponential function with the natural base e	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Guided and independent practice using the TI 83	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips
How does one identify and find an inverse of a function?	Define a one-to-one function Test for inverses graphically Write the inverse as a function of x	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Lecture and class discussion	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

How are the properties of logarithms used to solve equations?	Use the properties of logarithms to solve equations Solve equations involving logarithmic and exponential expressions	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Guided and independent practice Worksheets to reinforce the concepts	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips
What is the relationship between the six trigonometric functions, their graphs, and the unit circle?	Radian Measure Graph the six trigonometric functions and observe the domain, range, and period shown Graph a trigonometric function using transformations Use trigonometric identities to verify equations	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Complete the chapter study guides	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.9-12.F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.9-12.F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.9-12.F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.

Differentiation

Jig saw lesson: Student groups can create presentations on how to graph each one of the six trigonometric functions' parent graphs and their inverse, and present the group's work to the class.

Technology

The graphing calculators are useful tools when recognizing the patterns in graphs of functions.

College and Workplace Readiness

Medical professionals use sinusoidal graphs to determine the health of an individual on monitors.

AP Calculus BC - Unit 02

Unit 2: Limits and Continuity

Enduring Understandings:

Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:

How does the concept of a limit lead to a derivative?

How are derivatives used to analyze the behavior of a function?

Unit Goals:

1. Evaluate limits.
2. Determine graphical behavior and continuity.

Recommended Duration: 1.5 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
What are the graphical, numerical, and analytical ways to determine a limit of a function at a given value and its continuity at that point?	Definition of a limit, one sided limits, properties of limits	Textbook Graphing calculator Supplemental AP materials	Zeno's Paradox	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How can a limit be used to determine the graphical behavior of a function?	End behavior models	Textbook Graphing calculator Supplemental AP materials	Graphing calculator Exploration of graphs	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

What are the features of continuous functions? What are the characteristics of a discontinuous function?	Definition of continuity/types of discontinuity	Textbook Graphing calculator Supplemental AP materials	Teacher created worksheet Interactive white board presentation Class discussion	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Students can create games (bingo, flashcards, monopoly, etc.) to demonstrate their understanding classifying functions as continuous or discontinuous.

Technology

Teacher will use graphing software programs such as WinPlot and the graphing calculators to reinforce the concept of limits and how it leads to the slope of a tangent line.

College and Workplace Readiness

When working with compound interest and profit functions, a banker needs to understand the limiting behaviors of business models.

AP Calculus BC - Unit 03

Unit 3: Derivatives

Enduring Understandings:
Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:
How are derivatives used to analyze the behavior of functions?

Unit Goals:
1. Evaluate derivatives.
2. Find the equation of a tangent line.

Recommended Duration: 2 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How does the definition of a derivative lead to the analysis of graphs?	Apply the definition of a derivative Discuss the different notations of a derivative Graph a function and its derivative Find a right-hand and left-hand derivative of a function	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Lecture and class discussion	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

What are the conditions for differentiability?	Illustrate the four different instances where a derivative fails to exist Use the Intermediate Value and Mean Value Theorems to show how differentiability implies continuity	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Guided and independent practice worksheets to reinforce the concepts	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips
What are the various rules for differentiation and how are they applied?	Power rule (positive and negative integer powers) Constant multiple rule Sum and difference rule Product rule Quotient rule Chain rule Derivatives of transcendental functions Use second and higher order derivatives	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Remote responder quiz	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

How does the derivative relate to velocity and acceleration?	Define acceleration and velocity in terms of a derivative	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Odometer project	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips
How is implicit differentiation an important tool in finding derivatives?	Differentiate a multivariable function using implicit differentiation Define the process of implicit differentiation	Diagnostic test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentation Graphing calculators Dry erase boards Geometer's Sketchpad	Lecture and class discussion	Written tests and quizzes Worksheets Project assessments Responses to discussion questions Journal assessments Closure questions Exit slips

MA.9-12.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Students can create flashcards of the derivative graphs of the function graphs provided by the teacher. Students can then get into groups where they can match function graphs to their derivative graphs.

Technology

Calculus in Motion® in conjunction with can Geometer's Sketchpad® be used to understand what the graph of a derivative will look like.

College and Workplace Readiness

Biologists use the instantaneous rate of change at a point to analyze population growth rate.

AP Calculus BC - Unit 04

Unit 4: Applications of Derivatives

Enduring Understandings:
Limits are the underlying concept supporting physical applications that are embedded in many fields.

Essential Questions:
How is calculus used to solve optimization and related rate problems?

Unit Goals:
1. Solve optimization and related rate problems.
2. Analyze the behavior of a function.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How does one determine a function's maximum or minimum value?	Critical point, extreme values of a function, relative extrema, absolute extrema	Textbook Graphing calculator Supplemental AP materials	TI 83 exploration	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How does one determine relative extrema and intervals on which a function is increasing or decreasing?	Increasing function, decreasing function, first derivative test, Mean Value Theorem	Textbook Graphing calculator Supplemental AP materials	Matching graphs of derivatives and functions	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

How does one determine concavity?	Point of inflection, the second derivative to determine concavity, the second derivative test	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation/class discussion	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How are the rates of changing quantities related?	Applications involving changing quantities-related rates	Textbook Graphing calculator Supplemental AP materials	Tootsie roll pop activity	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How is the derivative used to optimize quantities?	Applications involving maximizing/minimizing quantities - optimization	Textbook Graphing calculator Supplemental AP materials	Build a box of maximum volume	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.
MA.9-12.F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Differentiation

Process differentiation: Students will self test using one of the practice tests available (internet based or pencil & paper).

Technology

The graphing calculator can be used to understand the behavior of functions.

College and Workplace Readiness

Agriculturalists use optimization to maximize/minimize dimensions of a field to be more efficient with their land usage.

AP Calculus BC - Unit 05

Unit 5: The Definite Integral

Enduring Understandings:
The Fundamental Theorem of Calculus connects differential and integral calculus.

Essential Questions:
How are integrals used to measure changing quantities?

Unit Goals:
1. Evaluate definite integrals.
2. Find the area under a curve.

Recommended Duration: 2 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can the definite integral be applied to finding area under a curve?	Approximation methods (LRAM, RRAM, MRAM, Trapezoidal)	Textbook Graphing calculator Supplemental AP materials	Bank robbery project	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How does estimating finite sums lay the foundation for finding the value of a definite integral?	Summation formulas evaluate a finite/infinite Riemann sum Properties of a definite integral Evaluate integrals using area	Textbook Graphing calculator Supplemental AP materials	Map project	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

How does the limit of a Reimann Sum relate to integration?	Definite integral, integrand, integrable function, apply the properties of a definite integral, evaluate a definite integral using area	Textbook Graphing calculator Supplemental AP materials	Odometer project part 2	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How does the Fundamental Theorem of Calculus connect integral and differential calculus?	Fundamental Theorem of Calculus Antiderivatives Indefinite integrals Mean Value (Average) Theorem for Integration	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation/class discussion	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.
MA.9-12.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.9-12.F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.

Differentiation

Content Differentiation: Discovery learning- The students are given a graph with a pre-drawn curve on the graph paper and are asked to develop a strategy to calculate the area between the curve and the x - axis using geometry.

Technology

Geometer's Sketchpad® is an excellent tool for student and/or teacher demonstrations of the essential components of integration.

College and Workplace Readiness

Scientists and actuaries use integration to analyze and calculate statistical data.

AP Calculus BC - Unit 06

Unit 6: Differential Equations

Enduring Understandings:

The Fundamental Theorem of Calculus connects differential and integral calculus.

Essential Questions:

How do differential equations help to describe rates of change?

Unit Goals:

1. Solve differential equations.

Recommended Duration: 2 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How are changing quantities described using differential equations?	Separable differentiable equations Indefinite integrals Slope fields	Textbook Graphing calculator Supplemental AP materials	Murder mystery problem Newton's Law of Cooling Spread of a disease	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
What is the relationship between slope fields and differential equations?	Slope fields Differential equations	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation/class discussion	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

What are some alternative techniques of integration?	Substitution Partial fractions Integration by parts Trigonometric substitution	Textbook Graphing calculator Supplemental AP materials	Cooperative learning using teacher created worksheets	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.
MA.9-12.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.9-12.F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.

Differentiation

Process Differentiation: Students can investigate the various techniques used for integration on internet tutorials sites and submit findings of their investigations for review.

Technology

The World Wide Web offers a variety of perspectives on techniques and applications of integration for students' enrichment.

College and Workplace Readiness

Meteorologists utilize slope fields in weather maps.

AP Calculus BC - Unit 07

Unit 7: Applications of the Definite Integral and Mathematical Modeling

Enduring Understandings:
Limits are the underlying concept supporting physical applications that are embedded in many fields.

Essential Questions:
How are area of bounded regions and volume of solids evaluated?

Unit Goals:
1. Find the area of a plane region and the volume of a solid.

Recommended Duration: 3 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can one estimate and calculate the area of the bounded region between two curves?	Rectangular approximation Definite integrals	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation/class discussion	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How can the definite integral be used to calculate the volume of a solid with a known cross section?	Area formulas Definite integrals	Textbook Graphing calculator Supplemental AP materials	Rice Krispies treat project	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

How can the definite integral be used to calculate the volume of a solid obtained by rotating a plane region?	Disc/washer method Shell method Definite integrals	Textbook Graphing calculator Supplemental AP materials	Model volume through candy kisses, cakes, ice cream cones	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.
MA.9-12.F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Differentiation

Product Differentiation: Students can create a solid and calculate the volume using each of the methods described above or research an application of the volume model and create a poster demonstrating appropriate methods for volume calculation.

Technology

The graphing calculators are utilized to represent or demonstrate an understanding of integration. Mathematica® software brings the three dimensional model to life.

College and Workplace Readiness

Using Calculus to determine areas and volumes of regions bounded by curves is utilized by mechanical engineers for dam and water tower designs and construction.

AP Calculus BC - Unit 08

Unit 8: L'Hopital's Rule

Enduring Understandings:
Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:
How does local linearity allow us to evaluate limits?

Unit Goals:
1. Evaluate limits for indeterminate form.

Recommended Duration: 2 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can limits of indeterminate form be evaluated using local linearity?	L'Hopital's rule	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation/class discussion	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How can one find the area of an unbounded region?	Improper integrals Direct comparison test P series	Textbook Graphing calculator Supplemental AP materials	Textbook concept worksheet	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Product differentiation: Students will partner to complete two problems using L'Hopital's rule. Then they will create an indeterminate equation of their own and help each other find the limit using L'Hopital's rule.

Technology

The graphing calculator will assist students in recognizing that a limit does exist for those functions of indeterminate form. They also offer a visual representation of various growth rates.

College and Workplace Readiness

Computer scientists use L'Hopital's rule when comparing the measures of 'same' growth rates.

AP Calculus BC - Unit 09

Unit 09: Infinite Series

Enduring Understandings:
Power series are an important tool for approximating and defining functions.
Limits are the underlying concept supporting physical applications that are embedded in many fields.

Essential Questions:
How does a Taylor Series represent a function?
How are transcendental functions described using infinite series?

Unit Goals:
1. Represent functions using Taylor Series.
2. Determine the interval of convergence.
3. Estimate the error bound when evaluating a function with a Taylor Polynomial.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How does one define a general power series and Taylor Series?	Infinite geometric series Definition of a Taylor Polynomial/Series Alternate methods of determining A Taylor Series from known series	Textbook Graphing calculator Supplemental AP materials	Technology activity with sine function and Taylor Series	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

How is the Taylor Series used to estimate functions?	La Grange error Alternating series error	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How can one determine the interval of convergence of a Taylor Series?	Ratio test Convergence tests	Textbook Graphing calculator Supplemental AP materials	AP review questions	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.9-12.F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.9-12.F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.9-12.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Differentiation

Product differentiation: Students should complete an independent project poster demonstrating the calculations of a perpetuity (giving $1000 a year to a charity forever with 8% interest – how much to invest), expected payoff (get $1.00 for fair toss of a coin every time it lands on heads) or the fractal of the equilateral triangle (sum of the areas removed from the triangle).

Technology

Web based videos offer visual representation of infinite series phenomena.

College and Workplace Readiness

Studies of atmospheric models use the calculus of polynomial expansions to express variables globally. Wind turbulence is described by equations that are solved using series approximations.

AP Calculus BC - Unit 10

Unit 10: Parametric, Vector and Polar Functions

Enduring Understandings:

Limits are the underlying concept supporting physical applications that are embedded in many fields.

Essential Questions:

How is motion analyzed using parametric equations and vectors?

How are slope and area measured in a polar coordinate system?

Unit Goals:

1. Use parametric equations to analyze motion with velocity and acceleration vectors.
2. Find areas of regions defined by polar equations.

Recommended Duration: 2 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How can one determine the slope of a parametrically defined curve?	Formula for slope of Parametric curves Speed formula	Textbook Graphing calculator Supplemental AP materials	Interactive white board presentation	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How can one define the slope of a polar defined curve?	Formula for slope of Polar curves Maximum functional Value of a polar curve	Textbook Graphing calculator Supplemental AP materials	TI 83 exploration	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

How does one determine the area of a polar defined region?	Area formula Interval of a polar Defined region	Textbook Graphing calculator Supplemental AP materials	Maximizing activity	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.

Differentiation

Process differentiation: Jig saw activity: Graphing exploration activity designed so that each student group is investigating slope of a different curve.

Technology

Science 360˚® offers the 'physics of sports' videos showing vectors and parametric equations in relations to sports.

College and Workplace Readiness

Physicists use calculus to represent change in motion.

AP Calculus BC - Unit 11

Unit 11: AP Review

Enduring Understandings:

Math can be communicated verbally, graphically, and using technology.

Essential Questions:

In what ways can the analytical work of calculus be supported?

How is a graphing calculator used as a problem solving tool?

Unit Goals:

1. Communicate mathematics in written form.

Recommended Duration: 4 weeks

Guiding/Topical Questions	Content/Themes/Skills	Resources and Materials	Suggested Strategies	Suggested Assessments
How does one use Calculus to justify applications?	Sign line, derivative tests, Mean Value Theorem, Fundamental Theorem of Calculus	Textbook Graphing calculator Supplemental AP materials	Graphing calculator exploration of limits, with use of tables	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation
How is a graphing calculator a helpful tool in calculus?	Use graphing calculator to evaluate derivatives, integrals, and graph functions	Textbook Graphing calculator Supplemental AP materials	Cooperative learning with previous AP questions	Tests (calculator/non-calculator) Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observation

MA.9-12.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
MA.9-12.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.9-12.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.9-12.F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.9-12.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.9-12.F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.9-12.F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.9-12.F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.9-12.F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.9-12.F-BF.1	Write a function that describes a relationship between two quantities.
MA.9-12.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.9-12.F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.9-12.F-BF.4	Find inverse functions.
MA.9-12.F-BF.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Differentiation

Process differentiation: Students will self test using one of the practice tests available (internet based or pencil & paper).

Technology

The graphing calculator, online testing and assistance are useful tools when preparing for the AP test.

College and Workplace Readiness

Calculus is an integral part of the products designed by engineers, scientists, economists, and actuarial analysts.