

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

OFFICE OF CURRICULUM AND INSTRUCTION

MATHEMATICS DEPARTMENT

AP CALCULUS AB

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE:

AUGUST 22, 2011

[SUPPORTING RESOURCES AVAILABLE IN DISTRICT RESOURCE SHARING](#)

APPENDIX A: ACCOMMODATIONS AND MODIFICATIONS

APPENDIX B: ASSESSMENT EVIDENCE

APPENDIX C: INTERDISCIPLINARY CONNECTIONS

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AP Calculus AB - Introduction

Introduction

Course Philosophy

Students will learn to appreciate the value of calculus as the mathematics of change and motion. This course will prepare students for further study in all branches of higher mathematics, science, and related fields. Calculus is not only the language for expressing physical laws in precise mathematical terms, but it is also a tool for studying these laws. The course emphasizes an approach to calculus that involves problems being expressed numerically, analytically, graphically, and verbally. Technology is used regularly to reinforce these approaches, to confirm written work, to implement experimentation, and assist in interpreting results.

Course Description

Calculus AB is primarily concerned with developing the students' understanding of the concepts of calculus and providing experiences with methods and applications. The course emphasizes a multi-representational approach to Calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are also important.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization, of an extensive taxonomy of functions, curves, theorems, or problem types. Although manipulation and computational competence is an important outcome, it is not the core of this course.

Technology will be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, applications, and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics.

Course Map and Proficiencies/Pacing

Course Map

| Relevant Standards | Enduring Understandings | Essential Questions | Assessments | | |
|--|---|--|-----------------------|--|--|
| | | | Diagnostic | Formative | Summative |
| MA.9-12.F-IF.4 MA.9-12.F-IF.5 MA.9-12.F-IF.7 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.1 | Analysis of the critical elements of functions is essential to calculus. | How are functions used in problem solving when modeling real world situations? How are trigonometric functions used to model real world situations? | Remote responder quiz | Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz | Unit test (multiple choice & free response questions) Alternative assessment Take home tests |
| MA.9-12.F-IF.1 MA.9-12.F-IF.2 MA.9-12.F-IF.4 MA.9-12.F-IF.7 | Functions can be analyzed graphically by their limiting behavior and rates of change. | How does the concept of a limit lead to a derivative? How are derivatives used to analyze the behavior of a function? How does local linearity allow us to evaluate a limit? | Remote responder quiz | Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz | Unit test (multiple choice & free response questions) Alternative assessment Take home tests |

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|---|---|---|------------------------------|---|---|
| <p>MA.9-12.F-IF.2 MA.9-12.F-IF.3 MA.9-12.F-BF.1 MA.9-12.F-BF.2 MA.9-12.F-LE.5</p> | <p>The Fundamental Theorem of Calculus connects differential and integral calculus.</p> | <p>How do differential equations describe rates of change? How are integrals used to measure changing quantities?</p> | <p>Remote responder quiz</p> | <p>Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz</p> | <p>Unit test (multiple choice & free response questions) Alternative assessment Take home tests</p> |
| <p>MA.9-12.F-IF.2 MA.9-12.F-IF.4 MA.9-12.F-IF.5 MA.9-12.F-BF.1 MA.9-12.F-BF.3</p> | <p>Limits are the underlying concept supporting physical applications that are imbedded in many fields.</p> | <p>How is calculus used to solve optimization and related rate problems? How are the areas of bounded regions and volume of solids evaluated?</p> | <p>Remote responder quiz</p> | <p>Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz</p> | <p>Unit test (multiple choice & free response questions) Alternative assessment Take home tests</p> |
| <p>MA.9-12.F-IF.1-9 MA.9-12.F-BF.1-5 MA.9-12.F-IF.7 MA.9-12.F-IF.8 MA.9-12.F-IF.9 MA.9-12.F-BF.1</p> | <p>Math can be communicated verbally, graphically, and using technology.</p> | <p>In what ways are the analytical work of calculus supported? How is a graphing calculator used as a problem solving tool?</p> | <p>Remote responder quiz</p> | <p>Do Now assignments Questions & answers Closure questions Group tests Remote responder quiz</p> | <p>Unit test (multiple choice & free response questions) Alternative assessment Take home tests</p> |

Proficiencies and Pacing

| Unit Title | Unit Understanding(s) and Goal(s) | Recommended Duration |
|-------------------------------------|---|----------------------|
| Unit 1: Functional Analysis | <p>Analysis of the critical elements of functions is essential to calculus.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Compare and contrast functions. 2. Use functions to model real life situations. | 2 weeks |
| Unit 2: Limits and Continuity | <p>Functions can be analyzed graphically by their limiting behavior and rates of change.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Evaluate limits. 2. Determine graphical behavior and continuity. | 2 weeks |
| Unit 3: Derivatives | <p>Functions can be analyzed graphically by their limiting behavior and rates of change.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Evaluate Derivatives 2. Find the equation of a tangent line. | 4 weeks |
| Unit 4: Applications of Derivatives | <p>Limits are the underlying concept supporting physical applications that are imbedded in many fields.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Solve optimization and related rate problems. 2. Analyze the behavior of a function. | 4 weeks |
| Unit 5: Definite Integrals | <p>The Fundamental Theorem of Calculus connects differential and integral calculus.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Evaluate definite integrals. 2. Find the area under a curve. | 2 weeks |
| Unit 6: Differential Equations | <p>The Fundamental Theorem of Calculus connects differential and integral calculus.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Solve differential equations. | 2 weeks |

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|--|---|----------------|
| <p>Unit 7: Applications of the Definite Integral and Mathematic Modeling</p> | <p>Limits are the underlying concept supporting physical applications that are imbedded in many fields.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Find the area and volume of a curve. | <p>4 weeks</p> |
| <p>Unit 8: L'Hopital's Rule</p> | <p>Limits are the underlying concept supporting physical applications that are imbedded in many fields.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Evaluate limits for indeterminate form. | <p>1 week</p> |
| <p>Unit 9: AP Review</p> | <p>Math can be communicated verbally, graphically, and using technology.</p> <p>At the conclusion of this unit, students will be able to:</p> <ol style="list-style-type: none"> 1. Communicate mathematics in written form. | <p>6 weeks</p> |

AP Calculus AB - Unit 01

Unit 1: Functional Analysis

Enduring Understandings:

Analysis of the critical elements of functions is essential to calculus.

Essential Questions:

How are functions used in problem solving when modeling real world situations?

How are trigonometric functions used to model real world situations?

Unit Goals:

1. Compare and contrast functions.
2. Use functions to model real life situations.

Recommended Duration: 2 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|---|---|--|---|---|
| How can you compare the graphs of piecewise, absolute value, and composite functions? | Define: function, domain, range, even function, odd function, and symmetry Graph examples of piecewise and absolute value functions Find the value of composite functions | Diagnostic Test Worksheets and samples Teacher driven worksheets Textbook ancillaries Multimedia presentations Graphing Calculators Dry erase boards Geometer's Sketchpad | Guided and independent practice using the graphing calculator | Written tests Quizzes Worksheets Project assessments Responses to discussion Questions Journal assessments Closure questions Exit slips |

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|--|---|---|--|--|
| <p>What are the differences between the models and graphs of exponential growth and exponential decay?</p> | <p>Represent the exponential function with base a</p> <p>Compare exponential growth and decay</p> <p>Represent the exponential function with the natural base e</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Guided and independent practice using the graphing calculator</p> | <p>Written tests Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |
| <p>How does one identify and find an inverse of a function?</p> | <p>Define a one-to-one function</p> <p>Test for inverses graphically</p> <p>Write the inverse as a function of x</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Lecture and class discussion</p> | <p>Written tests Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |

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| <p>How are the properties of logarithms used to solve equations?</p> | <p>Use the properties of logarithms to solve equations</p> <p>Solve equations involving logarithmic and exponential expressions</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Guided and independent practice worksheets to reinforce the concepts</p> | <p>Written tests Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |
| <p>What is the relationship between the six trigonometric functions, their graphs, and the unit circle?</p> | <p>Radian Measure</p> <p>Graph the six trig functions and observe the domain, range, and period shown</p> <p>Graph a trig function using transformations</p> <p>Use trigonometric identities to verify equations</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Complete the chapter study guides</p> | <p>Written tests Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

MA.9-12.F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MA.9-12.F-IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MA.9-12.F-IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

MA.9-12.F-BF.1

Write a function that describes a relationship between two quantities.

Differentiation

Jig saw lesson: Student groups can create presentations on how to graph each one of the six trigonometric functions' parent graphs and their inverse, and present the group's work to the class.

Technology

The graphing calculators are useful tools when recognizing the patterns in graphs of functions.

College and Workplace Readiness

Medical professionals use sinusoidal graphs to determine the health of an individual on monitors.

AP Calculus AB - Unit 02

Unit 2: Limits and Continuity

Enduring Understandings:

Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:

How does the concept of a limit lead to a derivative?

How do we use derivatives to analyze the behavior of a function?

Unit Goals:

1. Evaluate limits.
2. Determine graphical behavior and continuity.

Recommended Duration: 2 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|--|---|--|--|--|
| What are the graphical, numerical, and analytical ways to determine a limit of a function at a given value and its continuity at that point? | Definition of a limit, one sided limits, properties of limits | Textbook Graphing calculator Supplemental AP materials | Zeno's Paradox | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How can a limit be used to determine the graphical behavior of a function? | End behavior models | Textbook Graphing calculator Supplemental AP materials | Graphing calculator exploration of graphs | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

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|---|--|---|---|--|
| <p>What are the features of continuous functions? What are the characteristics of a discontinuous function?</p> | <p>Definition of continuity/types of discontinuity</p> | <p>Textbook Graphing calculator Supplemental AP materials</p> | <p>Teacher created worksheet Interactive white board presentation/class discussion</p> | <p>Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations</p> |
|---|--|---|---|--|

MA.9-12.F-IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Students can create games (bingo, flashcards, monopoly, etc.) to demonstrate their understanding of classifying functions as continuous or discontinuous.

Technology

Teacher will use graphing software programs such as WinPlot and the graphing calculators to reinforce the concept of limits and how it leads to the slope of a tangent line.

College and Workplace Readiness

When working with compound interest and profit functions, a banker needs to understand the limiting behaviors of business models.

AP Calculus AB - Unit 03

Unit 3: Derivatives

Enduring Understandings:

Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:

How do we use derivatives to analyze the behavior of functions?

Unit Goals:

1. Evaluate Derivatives.
2. Find the equation of a tangent line.

Recommended Duration: 4 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|---|--|---------------------------|------------------------------|-----------------------------------|
| How does the definition of a derivative lead to the analysis of graphs? | Apply the definition of a derivative Discuss the different notations of a derivative Graph a function and its derivative Find a right-hand and left-hand derivative of a function | Diagnostic Test | Lecture and class discussion | Written tests |
| | | Worksheets and samples | | Quizzes |
| | | Teacher driven worksheets | | Worksheets |
| | | Textbook ancillaries | | Project assessments |
| | | Multimedia presentations | | Responses to discussion questions |
| | | Graphing Calculators | | Journal assessments |
| | | Dry erase boards | | Closure questions |
| Geometer's Sketchpad | Exit slips | | | |

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| <p>What are the conditions for differentiability?</p> | <p>Illustrate the four different instances where a derivative fails to exist</p> <p>Use the Intermediate Value and Mean Value Theorems to show how differentiability implies continuity</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Guided and independent practice worksheets to reinforce the concepts</p> | <p>Written tests</p> <p>Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion Questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |
| <p>What are the various rules for differentiation and how are they applied?</p> | <p>Power Rule (positive and negative integer powers)</p> <p>Constant Multiple Rule</p> <p>Sum and Difference Rule</p> <p>Product Rule</p> <p>Quotient Rule</p> <p>Chain Rule</p> <p>Derivatives of transcendental functions</p> <p>Use second and higher order derivatives</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Remote responder quiz</p> | <p>Written tests</p> <p>Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |

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|--|--|---|-------------------------------------|---|
| <p>How does the derivative relate to velocity and acceleration?</p> | <p>Define acceleration and velocity in terms of a derivative</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Odometer Project</p> | <p>Written tests</p> <p>Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |
| <p>How is implicit differentiation an important tool in finding derivatives?</p> | <p>Differentiate a multivariable function using implicit differentiation</p> <p>Define the process of implicit differentiation</p> | <p>Diagnostic Test</p> <p>Worksheets and samples</p> <p>Teacher driven worksheets</p> <p>Textbook ancillaries</p> <p>Multimedia presentations</p> <p>Graphing Calculators</p> <p>Dry erase boards</p> <p>Geometer's Sketchpad</p> | <p>Lecture and class discussion</p> | <p>Written tests</p> <p>Quizzes</p> <p>Worksheets</p> <p>Project assessments</p> <p>Responses to discussion questions</p> <p>Journal assessments</p> <p>Closure questions</p> <p>Exit slips</p> |

| | |
|----------------|---|
| MA.9-12.F-IF.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$. |
| MA.9-12.F-IF.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| MA.9-12.F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. |
| MA.9-12.F-IF.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |

Differentiation

Students can create flashcards of the derivative graphs of the function graphs provided by the teacher. Students can then get into groups where they can match function graphs to their derivative graphs.

Technology

Calculus in Motion® in conjunction with Geometer's Sketchpad® can be used to understand what the graph of a derivative will look like.

College and Workplace Readiness

Biologists use the instantaneous rate of change at a point to analyze population growth rate.

AP Calculus AB - Unit 04

Unit 4: Applications of Derivatives

Enduring Understandings:

Limits are the underlying concept supporting physical applications that are imbedded in many fields.

Essential Questions:

How do we use calculus to solve optimization and related rate problems?

Unit Goals:

1. Solve optimization and related rate problems.
2. Analyze the behavior of a function.

Recommended Duration: 4 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|--|---|--|--|--|
| How does one determine a function's maximum or minimum value? | Critical point, extreme values of a function, relative extrema, absolute extrema | Textbook Graphing calculator Supplemental AP materials | Graphing calculator exploration | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How does one determine relative extrema and intervals on which a function is increasing or decreasing? | Increasing function, decreasing function, first derivative test, Mean Value Theorem | Textbook Graphing calculator Supplemental AP materials | Matching graphs of derivatives and functions | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

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|--|---|--|--|--|
| How does one determine concavity? | Point of inflection, the second derivative to determine concavity, the second derivative test | Textbook Graphing calculator Supplemental AP materials | Interactive white board presentation/class discussion | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How are the rates of changing quantities related? | Applications involving changing quantities- related rates | Textbook Graphing calculator Supplemental AP materials | Tootsie Roll Pop activity | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How is the derivative used to optimize quantities? | Applications involving maximizing/minimizing quantities - optimization | Textbook Graphing calculator Supplemental AP materials | Build a box of maximum volume | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

MA.9-12.F-BF.1

Write a function that describes a relationship between two quantities.

MA.9-12.F-BF.3

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Differentiation

Process differentiation: Students will self test using one of the practice tests available (internet based or pencil & paper).

Technology

The graphing calculator can be used to understand the behavior of functions.

College and Workplace Readiness

Agriculturalists use optimization to maximize/minimize dimensions of a field to be more efficient with their land usage.

AP Calculus AB - Unit 05

Unit 5: The Definite Integral

Enduring Understandings:

The Fundamental Theorem of Calculus connects differential and integral calculus.

Essential Questions:

How are integrals used to measure changing quantities?

Unit Goals:

1. Evaluate definite integrals.
2. Find the area under a curve.

Recommended Duration: 2 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|--|---|--|----------------------|--|
| How can the definite integral be applied to finding area under a curve? | Approximation methods (LRAM, RRAM, MRAM, trapezoidal) | Textbook Graphing calculator Supplemental AP materials | Bank Robbery Project | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How does estimating finite sums lay the foundation for finding the value of a definite integral? | Summation formulas evaluate a finite/infinite Riemann Sum Properties of a definite integral Evaluate integrals using area | Textbook Graphing calculator Supplemental AP materials | Map Project | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

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|--|---|--|---|--|
| How does the limit of a Riemann Sum relate to integration? | Definite integral, integrand, integral function, apply the properties of a definite integral, evaluate a definite integral using area | Textbook Graphing calculator Supplemental AP materials | Odometer Project Part 2 | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How does the Fundamental Theorem of Calculus connect integral and differential calculus? | Fundamental Theorem of Calculus Anti-derivatives Indefinite integrals Mean Value (Average) Theorem for integration | Textbook Graphing calculator Supplemental AP materials | Interactive white board presentation/class discussion | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

MA.9-12.F-IF.2
MA.9-12.F-IF.3
MA.9-12.F-BF.1
MA.9-12.F-BF.2
MA.9-12.F-LE.5

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
Write a function that describes a relationship between two quantities.
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Interpret the parameters in a linear or exponential function in terms of a context.

Differentiation

Content Differentiation: Discovery learning- The students are given a graph with a pre-drawn curve on the graph paper and are asked to develop a strategy to calculate the area between the curve and the x - axis using geometry.

Technology

Geometer's Sketchpad® is an excellent tool for student and/or teacher demonstrations of the essential components of integration.

College and Workplace Readiness

Scientists and actuaries use integration to analyze and calculate statistical data.

AP Calculus AB - Unit 06

Unit 6: Differential Equations

Enduring Understandings:

The Fundamental Theorem of Calculus connects differential and integral calculus.

Essential Questions:

How do differential equations describe rates of change?

Unit Goals:

1. Solve differential equations.

Recommended Duration: 2 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|---|--|--|--|--|
| How are changing quantities described using differential equations? | Separable differentiable equations Indefinite integrals Slope fields | Textbook Graphing calculator Supplemental AP materials | Murder Mystery Problem Newton's Law of Cooling Spread of a disease | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| What is the relationship between slope fields and differential equations? | Slope fields Differential equations | Textbook Graphing calculator Supplemental AP materials | Interactive white board presentation/class discussion | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

| | | | | |
|--|----------------------------|--|---|---|
| What are some alternative techniques of integration? | Substitution | Textbook Graphing calculator Supplemental AP materials | Cooperative learning using teacher created worksheets | Tests -calculator/non-calculator |
| | Partial fractions | | | Quizzes |
| | Integration by parts | | | Homework |
| | Trigonometric substitution | | | Cooperative learning sessions Alternative assessment Teacher observations |

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.3

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

MA.9-12.F-BF.1

Write a function that describes a relationship between two quantities.

MA.9-12.F-BF.2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

MA.9-12.F-LE.5

Interpret the parameters in a linear or exponential function in terms of a context.

Differentiation

Process Differentiation: Students can investigate the various techniques used for integration on internet tutorials sites and submit findings of their investigations for review.

Technology

The World Wide Web offers a variety of perspectives on techniques and applications of integration for students' enrichment.

College and Workplace Readiness

Meteorologists utilize slope fields in weather maps.

AP Calculus AB - Unit 07

Unit 7: Applications of the Definite Integral and Mathematical Modeling

Enduring Understandings:

Limits are the underlying concept supporting physical applications that are imbedded in many fields.

Essential Questions:

How are the areas of bounded regions and volume of solids be evaluated?

Unit Goals:

1. Find the area and volume of a curve.

Recommended Duration: 4 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|--|--|--|---|--|
| How can one estimate and calculate the area of the bounded region between two curves? | Estimating area under a curve using rectangles Definite integrals | Textbook Graphing calculator Supplemental AP materials | Interactive white board presentation/class discussion | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How can the definite integral be used to calculate the volume of a solid with a known cross section? | Area formulas Definite integrals | Textbook Graphing calculator Supplemental AP materials | Rice Krispies Treats project | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

| | | | | |
|---|--|--|---|--|
| How can the definite integral be used to calculate the volume of a solid obtained by rotating a plane region? | Disc/Washer Method Shell Method Definite integrals | Textbook Graphing calculator Supplemental AP materials | Model volume through candy kisses, cakes, ice cream cones | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
|---|--|--|---|--|

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

MA.9-12.F-BF.1

Write a function that describes a relationship between two quantities.

MA.9-12.F-BF.3

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Differentiation

Product Differentiation: Students can create a solid and calculate the volume using each of the methods described above or research an application of the volume model and create a poster demonstrating appropriate methods for volume calculation.

Technology

The graphing calculators are utilized to represent or demonstrate an understanding of integration. Mathematica® software brings the three dimensional model to life.

College and Workplace Readiness

Using Calculus to determine areas and volumes of regions bounded by curves is utilized by mechanical engineers for dam and water tower designs and construction.

AP Calculus AB - Unit 08

Unit 8: L'Hopital's Rule

Enduring Understandings:

Functions can be analyzed graphically by their limiting behavior and rates of change.

Essential Questions:

How is local linearity used to evaluate limits?

Unit Goals:

1. Evaluate limits for indeterminate form.

Recommended Duration: 1 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|--|-----------------------|--|---|--|
| How can limits of indeterminate form be evaluated? | L'Hopital's Rule | Textbook Graphing calculator Supplemental AP materials | Interactive white board presentation/class discussion | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

MA.9-12.F-IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

MA.9-12.F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.9-12.F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.9-12.F-IF.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Differentiation

Product differentiation: Students will partner to complete two problems using L'Hopital's Rule. Then they will create an indeterminate equation of their own and help each other find the limit using L'Hopital's Rule.

Technology

The graphing calculator will assist students in recognizing that a limit does exist for those functions of indeterminate form. They also offer a visual representation of various growth rates.

College and Workplace Readiness

Computer scientists use L'Hopital's Rule when comparing the measures of 'same' growth rates

AP Calculus AB - Unit 09

Unit 9: AP Review

Enduring Understandings:

Math can be communicated verbally, graphically, and using technology.

Essential Questions:

In what ways are the analytical work of calculus supported?

How is a graphing calculator used as a problem solving tool?

Unit Goals:

1. Communicate mathematics in written form.

Recommended Duration: 6 weeks

| Guiding/Topical Questions | Content/Themes/Skills | Resources and Materials | Suggested Strategies | Suggested Assessments |
|--|---|--|---|--|
| How does one use calculus to justify applications? | Derivative tests, Mean Value Theorem, Fundamental Theorem of Calculus | Textbook Graphing calculator Supplemental AP materials | Graphing calculator exploration of limits, with use of tables | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |
| How is a graphing calculator a helpful tool in calculus? | Use graphing calculator to evaluate derivatives, integrals, and graph functions | Textbook Graphing calculator Supplemental AP materials | Cooperative learning involving previous AP exams | Tests -calculator/non-calculator Quizzes Homework Cooperative learning sessions Alternative assessment Teacher observations |

| | |
|------------------|---|
| MA.9-12.F-IF.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$. |
| MA.9-12.F-IF.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| MA.9-12.F-IF.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. |
| MA.9-12.F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. |
| MA.9-12.F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. |
| MA.9-12.F-IF.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| MA.9-12.F-IF.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| MA.9-12.F-IF.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. |
| MA.9-12.F-IF.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| MA.9-12.F-BF.1 | Write a function that describes a relationship between two quantities. |
| MA.9-12.F-BF.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| MA.9-12.F-BF.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. |
| MA.9-12.F-BF.4 | Find inverse functions. |
| MA.9-12.F-BF.5 | Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. |
| MA.12.4.1.12 A.1 | Extend understanding of the number system to all real numbers. |

Differentiation

Process differentiation: Students will self test using one of the practice tests available (internet based or pencil & paper).

Technology

The graphing calculator, and online testing and assistance are useful tools when preparing for the AP test.

College and Workplace Readiness

Calculus is an integral part of the products designed by engineers, scientists, economists, and actuarial analysts.